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Should you show me the money? Concrete objects both hurt and help performance on mathematics problems

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Abstract

How do concrete objects that cue real-world knowledge affect students' performance on mathematics word problems? In Experiment 1, fourth- and sixth-grade students ($N = 229$) solved word problems involving money. Students in the experimental condition were given bills and coins to help them solve the problems, and students in the control condition were not. Students in the experimental condition solved fewer problems correctly. Experiment 2 tested whether this effect was due to the perceptually rich nature of the materials. Fifth-grade students ($N = 79$) were given: perceptually rich bills and coins, bland bills and coins, or no bills and coins. Students in the perceptually rich condition made the most errors; however, their errors were least likely to be conceptual errors. Results suggest that the use of perceptually rich concrete objects conveys both advantages and disadvantages in children's performance in school mathematics.

Keywords: Mathematics; Problem solving; Word problems; Symbols; Expertise reversal effect

1. Introduction

It is widely assumed that students perform better when mathematics problems are presented in the context of practical, real-world situations than when they are presented in traditional school contexts. This assumption is backed by number of theories. From a cognitive perspective, presenting material in the context of real-world situations should facilitate performance because it (a) provides cues for recall of effective problem-solving strategies (Kotovsky, Hayes, & Simon, 1985; Schliemann & Carraher, 2002), (b) reduces cognitive load by facilitating the chunking of task elements into more manageable pieces (Ericsson, Chase, & Faloon, 1980), (c) prevents students from treating the problems as artificial ‘‘games’’ that bear little relation to the world outside of school (Greer, 1997; Verschaffel, Greer, & De Corte, 2000), and (d) enables students to mentally simulate and ‘‘ground’’ material that might otherwise be too abstract to understand (Glenberg, Gutierrez, Levin, Japuntich, & Kaschak, 2004). This practice is even hypothesized to benefit non-cognitive aspects of problem-solving performance, such as students' motivation and interest in the task

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(Grigorenko, Jarvin, & Sternberg, 2002). In the present study, we focus on the cognitive aspects of activating students' real-world knowledge during problem solving.

The practice of activating students' real-world knowledge is based not only on theoretical deduction, but also on empirical data. Students have been shown to demonstrate more advanced mathematical reasoning in real-world settings (DeFranco & Curcio, 1997; Guberman, 1996; Resnick, 1987; Saxe, 1988; Taylor, 2006), and they also have been shown to perform better in academic settings when those settings appeal to real-world knowledge (Verschaffel et al., 2000; Wyndhamn & Säljö, 1997). A seminal study in this area is Carraher, Carraher, and Schliemann's (1985) *Mathematics in the Streets and in the Schools*. In the study, young vendors (ages 9–15 years) working in Brazil performed better on problems presented in the vending context (e.g., I'd like ten coconuts. How much is that?) than on the same problems presented symbolically (e.g., 35×10). Based on these findings, Carraher, Carraher, and Schliemann (1987) hypothesized that schoolchildren might also benefit from contexts designed to activate real-world knowledge. They performed a follow-up study in which Brazilian third graders solved problems in (a) a simulated-store context in which students sold items to the experimenter and the to-be-purchased items were laid out on the table, (b) a word-problem context in which the mathematics was embedded in a story, and (c) a symbolic context with no accompanying real-world context. Students performed better in the store and word-problem contexts than in the symbolic context, which suggests that activating real-world knowledge facilitates performance.

Baranes, Perry, and Stigler (1989) attempted to replicate these findings with U.S. third graders. However, they found that students did not generally perform better on word problems than on symbolic problems. Although these results conflicted with the findings of Carraher et al. (1985, 1987; see also Koedinger & Nathan, 2004), they were consistent with other research. Students around the world are notoriously bad at drawing on their real-world knowledge when solving word problems in school (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980; Reusser & Stebler, 1997; Verschaffel et al., 2000; Yoshida, Verschaffel, & De Corte, 1997). Instead, they tend to rely on algorithms and constraints learned in school, even if those constraints do not make sense when the given problem is interpreted in the context of the real world (Reusser & Stebler, 1997; Schoenfeld, 1989; Skemp, 1987; Verschaffel et al., 2000). For example, when presented with a problem in which the goal is to figure out how many balloons a specified number of children get, many students provide a decimal fraction as the answer (e.g., 4.5) even though it is unrealistic for each child to get only part of a balloon (Verschaffel, De Corte, & Lasure, 1994).

In light of these findings, many researchers have begun to look closely at how situations can be arranged to encourage students to draw on their real-world knowledge when solving mathematics problems in school (De Bock, Verschaffel, Janssens, Van Dooren, & Claes, 2003; Reusser & Stebler, 1997; Schliemann & Carraher, 2002; Taylor, 2006; Yoshida et al., 1997). One promising method involves immersing students in a classroom culture in which problems are treated as modeling activities in which students work together in groups with the teacher to understand the context, generate plausible contextually relevant approaches, and discuss the merits of different approaches (Gravemeijer, 2002; Greer, 1997; Verschaffel et al., 2000). This modeling activity has been shown to help students make sense of word problems (Verschaffel & De Corte, 1997). However, it takes considerable time and resources to implement, and it requires teachers to be open to dramatic changes in their classrooms (Greer, 1997). Thus, an equally important goal for researchers is to ascertain how situations can be arranged to help individual children succeed when problems are presented in more traditional school settings.

Baranes et al. (1989) hypothesized that a relatively simple contextual modification could push U.S. students to draw on their real-world knowledge when completing word problems. They varied the story context along with the degree to which the numbers used in the problems mapped onto that context. For example, multiples of 25 map onto a money context more easily than do multiples of 15 because 25 is the value of a quarter. Students in the study drew on their real-world knowledge and performed better on word problems (than on comparable symbolic problems) when problems involved money and multiples of 25. These findings suggest that it takes contextual support over and above what is typically present in school word problems to push students to use their real-world knowledge and improve their performance. However, the types of changes that need to be made for students to reap some benefits may be relatively minor and easy to implement.

One straightforward method for activating real-world knowledge when students are solving word problems is to provide them with supplemental materials, such as concrete objects, to reinforce the real-world scenarios described in the problems. For example, if a problem describes a situation in which items are being purchased or money is being divided among people (as in Baranes et al., 1989), then money could be made available to students as a cue. Teachers are often encouraged to use concrete objects during instruction to help students ground mathematical ideas (Burns, 1996; National Council of Teachers of Mathematics, 2000). The use of such objects may also help students perform to the best of their abilities by encouraging students to consult their real-world knowledge. Indeed, both Guberman (1996) and Saxe (1988) have shown that the presence of real currency facilitated Brazilian children's performance on mathematics problems. Thus, there is reason to predict that students will perform better on word problems when they have access to concrete objects that cue real-world knowledge than when they do not have access to such objects (Hiebert & Wearne, 1996; Hiebert, Wearne, & Taber, 1991).

The role and relevance of real-world knowledge in solving mathematics problems brings up an important issue of the design of concrete materials used in school. Although teachers cannot always use authentic artifacts such as real currency, many companies (e.g., Eric Armin Inc. [EAI] Education, Learning Resources, etc.) sell "classroom money kits" containing bills and coins that resemble real currency. These bills and coins are often used in the classroom to support mathematical concepts involved in saving money, or buying and selling goods. Producers of these kits often go to great lengths to ensure that the kits are as realistic as possible, including detailed patterning and inscriptions on bills and coins. The motivating assumption seems to be that making the objects highly realistic will help to evoke students' real-world knowledge, and thus, help them perform better than they otherwise would on relevant problems.

Although this reasoning is consistent with the aforementioned evidence, there are also reasons to think twice about using concrete objects that are perceptually rich and highly realistic. One challenge, for example, may be for students to think about more general properties that are not tied to the specific characteristics of the individual objects. In the case of the bills and coins in classroom money kits, students need to interpret mathematics problems not only as representing the exchange of specific bills and coins but also the relation of a particular money problem to the arithmetic algorithms learned in school. Several lines of research and theory (Bassok & Holyoak, 1989; Goldstone & Sakamoto, 2003; Sloutsky, Kaminski, & Heckler, 2005; Sweller, 1994; Uttal, Scudder, & DeLoache, 1997) suggest that this sort of transfer and generalization of knowledge may be *harmed* when stimuli are perceptually rich.

For example, Goldstone and Sakamoto (2003) found that undergraduates who learned ecology principles from a perceptually attractive computer display actually had more difficulty than those who learned from a bland display transferring their knowledge to a problem in a new domain that was perceptually dissimilar but conceptually similar to what they had learned. Similarly, Kaminski and Sloutsky (2007) found negative effects of perceptually rich symbols in their work on students' understanding of an abstract mathematical concept. Such findings are consistent with the hypothesis that the surface features of a symbol matter (Gravemeijer, 2002; Schnotz & Bannert, 2003). Perceptually rich, realistic surface features may initially activate real-world knowledge and help problem solvers construct contextually relevant interpretations of a problem. Yet, such features become redundant once the corresponding real-world knowledge has been activated (van Gog, Paas, & van Merrienboer, 2008; Kalyuga, Ayres, Chandler, & Sweller, 2003). Because redundant information is difficult for problem solvers to ignore, it imposes extraneous cognitive load (Kalyuga et al., 2003). Thus, problem solvers' attention may be drawn toward the redundant surface features of a symbol (e.g., color, size, texture) and away from the abstract ideas being represented by the symbol. Because of these consequences, Gravemeijer recommends gradually "decontextualizing" symbols used in classrooms (cf. Lehrer & Schauble, 2002). This decontextualization may be necessary for identifying the relevant and essential aspects of a given problem (Goldstone, 2006).

In addition to highlighting certain aspects of symbols while deemphasizing others, the use of highly concrete and perceptually rich objects also may hinder students' performance because it may require students to change their representations of familiar objects. The process of changing may be difficult, especially if students' old way of representing the objects is well established. For example, if children are very familiar with using play money as a toy (e.g., board games such as Monopoly), it may be difficult for them to view the play money as something that can help them solve problems in school (cf. De Bock et al., 2003; DeLoache, 1995; Uttal, Liu, & DeLoache, 2006; Uttal, Marulis, & Lewis, 2007). Once individuals represent a concept one way, it can be difficult for them to let go of that representation so they can represent it in a new way (Mack, 1995; McNeil & Alibali, 2005; Son & Goldstone, submitted for publication).

Interestingly, many of the best-known and most frequently used concrete objects in mathematics classrooms are rather bland. For example, Base-10 blocks are simply uniform-colored blocks, with different sizes representing the different units of tens, and Digi-blocks are a uniform color. Similarly, many of the Montessori materials (e.g., the Pink Tower) are not particularly interesting as objects in their own right, perhaps thereby allowing students to focus more on what the materials are designed to help them discover (e.g., concepts of ratio and proportion, Lillard, 2005). Put simply, at least in some situations, concrete objects may not need to be perceptually rich to help students solve problems; in fact, the opposite may sometimes be true (Cai, 1995; Stevenson & Stigler, 1992).

Thus, there are reasons both to support and to question the practice of providing students with concrete objects that are perceptually rich and highly realistic during problem solving. On one hand, the presence of these objects may activate real-world knowledge, which may ground abstract concepts and provide support for ‘‘sense making’’ and informal reasoning strategies. In this case, the presence of such objects should improve understanding of the mathematical concepts involved in a given problem. On the other hand, perceptual richness could potentially draw students’ attention to features of the objects themselves instead of to the mathematics at hand. In this case, the presence of the objects should be associated with more error prone problem solving. To our knowledge, no studies have tested these hypotheses directly.

1.1. Aims – hypotheses

In the present study, we investigated whether concrete objects that are perceptually rich and highly realistic convey an advantage over (a) no objects and (b) perceptually bland objects in children’s performance on word problems. In Experiment 1, we investigated whether the presence of perceptually rich, highly realistic bills and coins improve students’ performance on money-relevant word problems. If children benefit from having their real-world knowledge of money activated, then they should perform better in the presence (vs. absence) of the bills and coins. However, if perceptually rich objects detract students’ attention from the mathematical ideas being represented, then children should perform worse in the presence (vs. absence) of the bills and coins. In Experiment 2, we varied the level of perceptual richness of the bills and coins to examine the effect on performance, and we considered whether perceptually rich bills and coins facilitate informal reasoning strategies, thus reducing students’ likelihood of making conceptual errors.

2. Experiment 1

2.1. Method

2.1.1. Participants

Participants were 113 fourth-grade students (9–10-year olds; 54 boys, 59 girls; 5% Asian, 18% Black, 18% Hispanic or Latino, 59% White) and 117 sixth-grade students (11–12-year olds; 57 boys, 60 girls; 2% Asian, 5% Black, 8% Hispanic or Latino, 85% White) recruited from public middle schools in Connecticut, USA. Approximately 30% of students received free or reduced-price lunch. One sixth-grade student was excluded because she did not follow instructions.

2.1.2. Tasks

To extend previous research (Carraher et al., 1985, 1987; Baranes et al., 1989), we focused on students’ performance on word problems involving money. The problems were a subset of a paper-and-pencil assessment designed to assess students’ understanding of concepts and procedures targeted in the National Council of Teachers of Mathematics (NCTM, 2000) Standards for Grades 3–5. We first pilot-tested several sets of problems and selected a final set of 10 problems to represent difficulty levels from easy to hard (difficulty level was inferred based on the percentage of pilot students who solved each problem correctly: 81% solved the easiest problem correctly and 11% solved the hardest problem correctly). Two representative examples are the following:

> *Problem #6*: ‘‘Charles buys a gumball for \$0.20 and a candy bar for \$1.15. If he gives the cashier \$5.00, how much change does he get back?’’ (medium level)
> *Problem #1*: ‘‘Maggie is trying to save money to buy a new CD player. She gets \$15.00 allowance at the end of each week. The CD player costs \$95.99. How many weeks will it take her to save the money she needs if she saves her whole allowance each week?’’ (medium level)

Students’ solutions were coded as correct or not correct. We summed the number of correct solutions to the 10 problems to obtain a total score. Because students’ writing was sometimes difficult to read, a second coder evaluated the solutions of a randomly selected 20% of the sample (46 students, 460 solutions). Agreement between coders was 94%.

2.1.3. Procedure

Students were randomly assigned to one of two conditions. In the experimental condition, students were given a stack of bills (major denominations up to $50) and a small plastic bag containing pennies, nickels, dimes, and quarters. The bills and coins were designed to activate students' real-world knowledge of U.S. currency, and they looked similar to real money (see Fig. 1, left panel). Students were told that they should use the bills and coins to help them solve the problems. However, we did not prescribe ways for students to use the materials because one of the major criticisms of manipulatives is that students are often required to handle them according to rules designated by the teacher, and this practice deters students from constructing their own understandings (Gravemeijer, 2002). We wanted students to use the money in whatever way made the most sense to them. Students in the experimental condition also had the paper-and-pencil test in front of them at all times, so they were able to use both bills and coins and traditional written strategies. We hoped that students would use the bills and coins as tools for problem solving, but our primary goal was to examine how their presence would affect students' performance. In the control condition, students solved the same word problems involving money, but without the presence of bills and coins.

Experimenters observed students solving the problems and noted whether those in the experimental condition used the money when solving the problems. Because children were tested in a large group setting, it unfortunately was not possible for experimenters to monitor each child's use of the bills and coins on a problem-by-problem basis. However, experimenters were assigned to monitor specific children throughout the testing session (approximately 5–8 children per experimenter). They checked on these children several times throughout the session and noted whether or not the children were interacting with the bills and coins (e.g., looking, touching, counting, etc.). Most children in the experimental condition (71%) were seen interacting with the bills and coins on a consistent basis.

To illustrate how students could use the bills and coins, consider Problem #6 in which Charles buys a gumball and a candy bar for $0.20 and $1.15 and gets back change from $5.00. To calculate the correct change using the bills and coins, students could have placed three one-dollar bills on the table, along with two dollars worth of change (e.g., four quarters, eight dimes, and four nickels). Then, they could take away two dimes (to represent the $0.20) and one dollar, one dime, and one nickel away (to represent the $1.15). Finally, they could count the money left on the table ($3.65). In

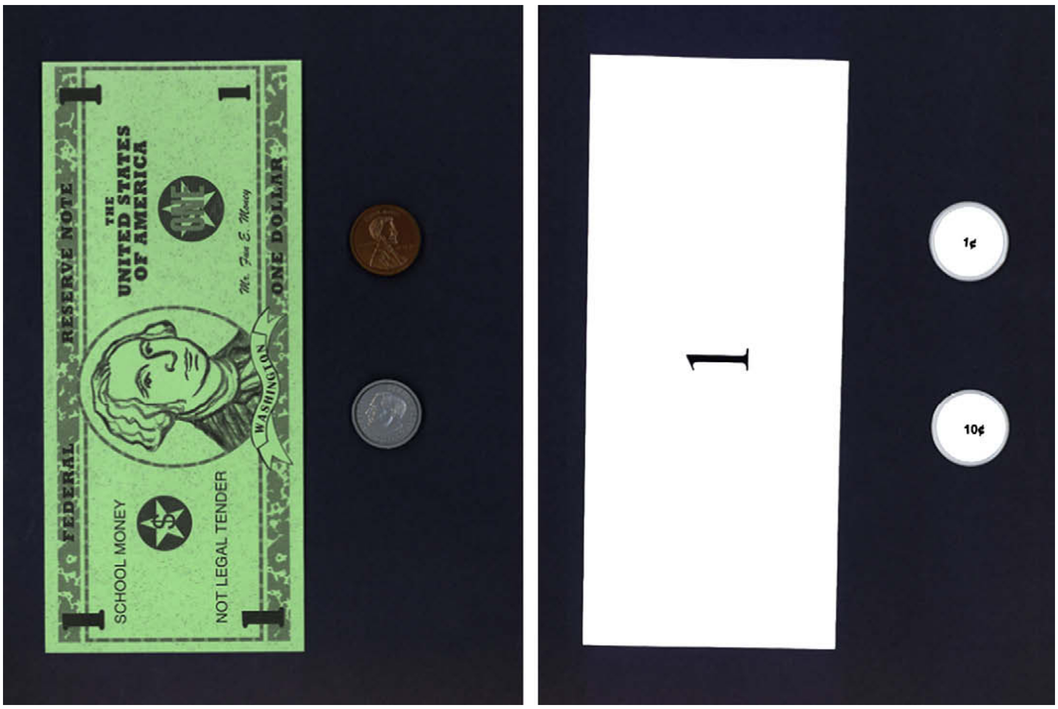

Fig. 1. Examples of the "perceptually rich" bills and coins used in Experiments 1 and 2 (left panel) and the "bland" bills and coins used in Experiment 2 (right panel).

the control condition, students typically used a two-step written procedure, such as adding \$0.20 to \$1.15 in column format to get \$1.35, and then subtracting \$1.35 from \$5.00 using a multidigit subtraction procedure to get \$3.65.

2.2. Results – discussion

We conducted a 2 (condition: experimental and control) × 2 (grade: fourth and sixth) ANOVA with number correct (out of 10) as the dependent measure. There was a significant main effect of condition, $F(1, 225) = 4.08$, $p = 0.04$, partial $\eta^2 = 0.02$. Students in the control condition solved more problems correctly ($M = 4.85$, SD = 3.11) than students in the experimental condition ($M = 4.26$, SD = 2.93). There was also a significant main effect of grade, $F(1, 225) = 93.24$, $p < 0.001$, partial $\eta^2 = 0.29$. Not surprisingly, sixth graders solved more problems correctly ($M = 6.15$, SD = 2.55) than fourth graders ($M = 2.91$, SD = 2.57). The interaction was not significant, $F(1, 225) = 0.02$, $p = 0.90$.

These results suggest that concrete objects designed to activate students' real-world knowledge may hinder performance on mathematics problems. However, because the effect size was small, the result needs to be replicated before conclusions can be drawn. When evaluating the effect size, it is important to note that the participant sample was diverse. This diversity is good for external validity, but large individual differences also contribute to a smaller effect size. We, therefore, worked with a more homogeneous sample in Experiment 2.

3. Experiment 2

The findings thus far are consistent with the suggestion that perceptually rich stimuli detract students' attention from the mathematical ideas being represented (Goldstone, 2006; Sloutsky et al., 2005). Classroom money kits contain bills and coins that are brightly colored and detailed in appearance. Although these characteristics are meant to activate students' real-world knowledge of U.S. currency, they may be counterproductive. Specifically, these characteristics may draw students' attention to the bills and coins themselves and away from their purpose as representations of monetary values (Uttal, 2003; Uttal et al., 1997). Also, because the bills and coins resemble play money, they may activate students' knowledge of play activities, which may hinder performance on school activities. Taken together, these accounts lead to the hypothesis that students should not perform as poorly when they use bills and coins that are not as perceptually rich and realistic looking. We tested this hypothesis in Experiment 2.

In addition, although the results of Experiment 1 suggest that the perceptually rich, realistic bills and coins hinder performance, it is still possible that they helped students in ways that transcended the accuracy of their solutions. Because the bills and coins are designed to activate real-world knowledge, they may promote conceptual understanding of the problems. Therefore, the errors of students in the control condition may have been more conceptually based than the errors made by students who had bills and coins. For example, consider Problem #6 in which students had to calculate how much change Charles would get back from five dollars if his items cost \$0.20 and \$1.15. Students in the control condition may have been making conceptual errors that reflect fundamental misunderstandings of the problems (e.g., writing \$5.00 + \$0.20 + \$1.15 = \$6.35), whereas students in the experimental condition may have been making arithmetic errors (e.g., writing \$5.00 − \$0.20 − \$1.15 = \$4.15). It was difficult to examine this hypothesis in Experiment 1 because students did not always show how they arrived at their answers. They often wrote answers with no accompanying written work. These "answer only" responses were more common in the experimental condition ($M = 9.24$, SD = 0.44) than in the control condition ($M = 7.48$, SD = 0.45), $F(1, 227) = 7.93$, $p = 0.01$, partial $\eta^2 = 0.29$. This finding is not surprising, given that students in the experimental condition were able to use the objects to work out the problems, and students were not given explicit instructions to show their work. "Answer only" responses were common across both conditions. They accounted for 42% of the responses overall, and 97% of students gave an "answer only" response on at least one problem. We presumed that the high number of "answer only" responses occurred because students were not told to show their work. We, therefore, asked students to show their work in Experiment 2, so we could get more specific information about how the conditions influenced students' thinking.

3.1. Method

3.1.1. Participants

Participants were 85 fifth-grade students (10–11 year olds; 47 boys, 38 girls; <1% Asian, 3% Black, 2% Hispanic or Latino, and 94% White) recruited from a public middle school in Connecticut, USA. Approximately, 10% of students received free or reduced-price lunch. Six students (5 boys, 1 girl) were excluded from the analyses because they did not follow instructions (e.g., did not solve the problems, talked too much during testing, etc.).

3.1.2. Tasks – procedure

The procedure was identical to that in Experiment 1 with three exceptions. First, the 10-word problems were presented on their own rather than as a part of a larger assessment. Second, prior to testing, all students were told to ''show their work'' on every problem. The specific instructions were ''Write your answer and whatever calculations you made to find that answer. For example, if you added two numbers, write that down, like $2+2=4$. It is very important for us to understand HOW you found your answer, so please be sure to **show all of your work** [bold in actual text].'' In addition to these instructions, the reminder phrase ''Show your work!'' was appended to the end of every problem. This emphasis on ''showing work'' was added so that we would be able to examine differences in the types of errors made by students in each condition. Students were not told that they had to show their work in any prescribed way, so they were free to show their work in ways that made sense to them. Many children showed their work by translating their actions with the bills and coins into mathematical symbols (e.g., one dollar plus one dime would be written as $1 + 10¢). However, some translated their actions into words (e.g., ''dollar plus dime''). Few children drew schematic representations of the bills and coins. Third, students were randomly assigned to one of *three* conditions. Two of the conditions were identical to the conditions in Experiment 1, and the third condition was added so we could examine the effect of making the objects less perceptually rich and less realistic in appearance. Thus, the three conditions were (1) the perceptually rich condition, in which bills and coins were identical to the bills and coins used in Experiment 1; (2) the bland condition, in which bills and coins were black-on-white bills and coins that were stripped of extraneous perceptual details; and (3) the control condition as in Experiment 1. Fig. 1 illustrates the differences between the two types of bills and coins. The bland condition was identical to the perceptually rich condition in all respects except for the actual appearance of the bills and coins. Children in both conditions were instructed to use the bills and coins to help them solve the problems (as in Experiment 1), and children in all conditions were asked to show their work.

3.1.3. Coding

Students' solutions were counted as ''correct'' only if the correct answer was provided along with appropriate accompanying written work. Because the importance of ''showing work'' was stressed prior to testing, ''answer only'' solutions accounted for only a small percentage (9%) of responses (compared with 42% in Experiment 1). A second coder again evaluated the solutions of a randomly selected 20% of the sample. Agreement for coding correctness was 96%.

Students' errors were further coded as conceptual or non-conceptual. Conceptual errors were defined as errors that demonstrate a conceptual misunderstanding about the problem (as in Koedinger & Nathan, 2004). For example, consider Problem #6 in which students had to calculate how much change Charles would get back from five dollars if his items cost $0.20 and $1.15. One conceptual error was to add up all the numbers in the problem (i.e., $5.00 + $0.20 + $1.15 = $6.35). Another conceptual error was to write the total amount paid ($1.35), rather than the total amount of change. Another conceptual error was to subtract $0.20 from $1.15 to get $0.95. Conceptual errors, such as these, were coded in contrast to other types of errors such as copy slips (student sets up the solution correctly but copies numbers incorrectly), arithmetic errors (student sets up the solution correctly but adds, subtracts, multiplies, or divides incorrectly), answer only (student provides the incorrect or correct answer without any accompanying work), and no response (only 1% of responses). We established reliability for the distinction between conceptual and non-conceptual errors by having two independent coders code students' errors; agreement was 92%. The majority of students' errors (80%) were either conceptual or arithmetic errors. Error codes were not treated as mutually exclusive categories. That is, some students made both a conceptual and an arithmetic error on the same problem, and their responses were coded accordingly. Errors and correct solutions were mutually exclusive categories (e.g., a solution could not be counted as a conceptual error if it was counted as correct).

3.2. Results – discussion

An ANOVA with condition (perceptually rich, bland, control) as the independent variable and number correct (out of 10) as the dependent measure revealed a significant main effect of condition, $F(2, 76) = 3.17$, $p = 0.048$, partial

$\eta^2 = 0.08$. To test our hypothesis that perceptually rich bills and coins hinder performance, we performed a set of orthogonal contrasts using Helmert coefficients. Consistent with the results of Experiment 1, perceptually rich bills and coins hindered performance on the problems. Students in the perceptually rich condition solved fewer problems correctly ($M = 4.08$, SD = 2.29) than students in the other two conditions ($M = 5.50$, SD = 2.34), $F(1, 76) = 6.34$, $p = 0.01$, partial $\eta^2 = 0.08$. The number of problems solved correctly by students in the bland condition ($M = 5.56$, SD = 2.20) did not differ significantly from the number of problems solved correctly by students in the control condition ($M = 5.45$, SD = 2.49), $F(1, 76) = 0.03$, $p = 0.86$. Thus, it was not the presence of objects per se that hindered performance on the problems.

We also examined performance on a problem-by-problem basis to make sure that the observed differences between conditions were not driven by any one problem, in particular. As shown in Table 1, the patterns observed on a problem-by-problem basis were consistent with the results of the overall analysis. The percentage of students who solved any given problem correctly was the lowest (or approximately tied for lowest) in the perceptually rich condition across all problems.

To gain a deeper understanding of whether the conditions affected the way students approached the problems, we performed a detailed analysis of students' written work on Problem #1 in which Maggie is saving her $15.00 weekly allowance to buy a new CD player that costs $95.99 and needs to calculate how many weeks it will take to buy it. We chose to examine students' work on this problem because (a) it showed one of the largest discrepancies in terms of correctness between students in the perceptually rich condition and students in the other two conditions (see Table 1), (b) students showed a good deal of work on the problem, and (c) there were systematic individual differences in the way students approached the problem.

One of the systematic differences from student-to-student on this problem was whether, or not, the student used a repeated addition strategy vs. a multiplication or division strategy. That is, some students added $15.00 in column format repeatedly until they reached the desired amount, whereas other students tried to incorporate multiplication or division into their approach. Students who used the repeated addition strategy ($N = 40$) were more likely to solve the problem correctly than were students who tried to incorporate multiplication or division into their approach ($N = 35$), 65% vs. 40%, $\chi^2(1, N = 75) = 4.69$, $p = 0.03$. Although the (more successful) repeated addition strategy was used by about 53% of children overall, it was used by only 18% of students in the perceptually rich condition. Students in the perceptually rich condition were more likely to try to incorporate multiplication or division into their approach than were students in the other conditions, $\chi^2(2, N = 75) = 15.77$, $p < 0.001$. This finding was not predicted a priori, so it should be interpreted with caution. However, it is consistent with the prediction that the presence of the perceptually rich bills and coins influences how students approach the problems.

3.2.1. Conceptual errors

Results thus far are consistent with Experiment 1. The presence of perceptually rich bills and coins hindered performance on word problems involving money. However, the perceptually rich bills and coins may have facilitated other aspects of performance. As evident in the analysis of student work on Problem #1 in which Maggie is saving for a CD player, the presence or absence of bills and coins may affect how students approach problems. Because they are designed to cue real-world knowledge, the perceptually rich bills and coins may have helped students conceptualize the problems. To address this hypothesis, we need to look beyond "correctness". Another way to look at the data is to examine the types of errors made by students in each condition. Of particular interest is the proportion of errors that were *conceptual* errors because these errors reflect conceptual misunderstandings about the problems.

We conducted an ANOVA with condition (perceptually rich, bland, control) as independent variable and proportion of errors that were conceptual errors as the dependent measure. Three students were excluded from this analysis

Table 1
Percentage of students who solved each problem (1–10) correctly in each condition in Experiment 2

Condition	Problem									
	1	2	3	4	5	6	7	8	9	10
Perceptually rich	36	16	36	72	32	72	48	28	32	36
Bland	64	24	52	72	40	80	76	36	52	60
Control	59	28	55	72	31	79	83	45	52	41

because they did not make errors. There was a significant effect of condition, $F(2, 73) = 3.38$, $p = 0.04$, partial $\eta^2 = 0.09$. To test our hypothesis that students in the perceptually rich condition may have a smaller proportion of errors that were conceptual errors than in the other two conditions, we performed a set of orthogonal contrasts using Helmert coefficients. Similar to the results of Experiment 1, performance in the perceptually rich condition was significantly different from performance in the other two conditions. However, in this case, the performance of students in the perceptually rich condition was ‘‘better’’ than the performance of students in the other two conditions. The proportion of errors that were *conceptual* errors was smaller for students in the perceptually rich condition ($M = 0.37$, SD = 0.24) than it was for students in the other two conditions combined ($M = 0.51$, SD = 0.26), $F(1, 73) = 5.28$, $p = 0.02$, partial $\eta^2 = 0.07$. The proportion of errors that were conceptual errors was lower for students in the bland condition ($M = 0.47$, SD = 0.23) than it was for students in the control condition ($M = 0.55$, SD = 0.29), but this difference was not statistically significant, $F(1, 73) = 1.21$, $p = 0.28$. Thus, when errors were made, they were less likely to be conceptual errors when students were given perceptually rich bills and coins than when students were given bland bills and coins or no bills and coins. However, it is important to keep in mind that the total number of errors was greater in the perceptually rich condition.

We again examined performance on a problem-by-problem basis to ensure that the observed differences between conditions were not driven by any one problem, in particular. As shown in Table 2, the patterns observed on a problem-by-problem basis were consistent with the overall analysis. Of the students who made an error on a given problem, the percentage of those who made a conceptual error vs. a non-conceptual error was the lowest (or approximately tied for lowest) in the perceptually rich condition across all problems except Problem #9, which seemed to be particularly difficult for students in the perceptually rich condition (see Table 1).

To gain a deeper understanding of how the conditions affected students' thinking, we analyzed answers given by students who made conceptual errors on Problem #8: ‘‘Daniel and Helen want to buy a birthday cake for their friend. Daniel has $6.00 and can spend ¼ of his money. Helen has $10.00 and can spend ¼ of her money. How much can they spend altogether?’’ We examined students' answers on this problem because it was one of the most difficult problems for students, and we noticed that some students appeared to have a ‘‘suspension of sense making’’ on it (Verschaffel et al., 2000). Specifically, some students provided an answer that was equal to or greater than $8. We considered this to be a ‘‘suspension of sense making’’ because students should be able to realize that the answer is less than $8 because $8 is the amount that would result if Daniel and Helen were able to spend half of their money. This type of ‘‘suspension of sense making’’ occurred more often in the control condition. Ten of the 12 students who made a conceptual error on this problem in the control condition put $8 or more for their answer, compared with one of eight students in the bland condition and three of seven students in the perceptually rich condition, $\chi^2(2, N = 27) = 9.95$, $p = 0.01$. Thus, the conceptual errors made by students in the control condition may have been more nonsensical than the conceptual errors made by students in the conditions with bills and coins. Taken together with the previous results, this finding provides additional support for the notion that the presence or absence of bills and coins affected how students thought about and approached the problems.

4. General discussion

We investigated how the presence of objects designed to activate real-world knowledge affects fourth- through sixth-grade students' performance on mathematics word problems. We exploited the fact that objects can differ in terms of their level of perceptual detail and resemblance to artifacts in the real world. For example, a plastic disk that is made to resemble a dime is more perceptually rich and similar to everyday objects, but a plain white disk with a black ‘‘10’’ printed on it is more bland and detached from specific referents. Students who were given perceptually rich,

Table 2
Percentage of erring students on each problem (1–10) who made a conceptual error in each condition in Experiment 2

Condition	Problem									
	1	2	3	4	5	6	7	8	9	10
Perceptually rich	56	57	37	29	0	29	0	40	59	25
Bland	89	79	53	71	0	40	50	38	25	70
Control	75	86	54	50	25	33	100	81	43	47

realistic bills and coins made more errors on word problems involving money than did students who were given bland bills and coins or no bills and coins. However, when students in the perceptually rich condition made errors, the errors were less likely to be conceptual errors.

Our findings support Gravemeijer's (2002) hypothesis that the superficial properties of inscriptions and symbols matter and suggest that the complementary nature of objects that are more vs. less perceptually rich and realistic may lead to trade-offs in mathematical problem-solving performance. Specifically, they may have different advantages and disadvantages for focusing students' attention on informal understanding vs. school-learned algorithms. In the following sections, we consider the potential processes involved in these trade-offs and highlight potential educational implications. We also discuss limitations and future directions.

4.1. Potential trade-offs between informal understanding and school-learned algorithms

As argued by Son and Goldstone (submitted for publication), activities that are designed to activate students' concrete, real-world knowledge are neither good nor bad in themselves, but rather lead to trade-offs in students' understanding of a problem by drawing attention toward certain ideas and away from others (cf. Gravemeijer, 2002). According to Carraher et al. (1987), when students are placed in meaningful contexts that activate their real-world knowledge of mathematics (e.g., a market), their attention is drawn toward the physical quantities involved (e.g., the objects being bought or sold, the money) and the types of operations being performed on those physical quantities (e.g., buying or selling). This focus allows students to capitalize on their informal reasoning skills and generate sensible solutions (Baranes et al., 1989; Carraher et al., 1985). In the present study, we saw this phenomenon reflected in the *types* of errors students made (and did not make) when they were given perceptually rich, realistic bills and coins. Specifically, the proportion of errors that were conceptual errors was lower in the perceptually rich condition than it was in the other conditions. We also saw it reflected in the nonsensical answers provided by children in the control condition on Problem #8 in which Daniel and Helen could each spend $\frac{1}{4}$ of their money to buy a birthday cake. Students were more likely to ignore applicable aspects of their real-world knowledge when they did not have access to bills and coins.

At the same time, we exposed a potential downside to activating real-world knowledge with concrete objects in performance settings – students may make a greater number of errors overall, especially in setting up and carrying out school-learned algorithms. In order for students to use concrete objects to represent mathematics, they need to go beyond their representations of the physical objects themselves. They need to draw analogies between the objects and the arithmetic algorithms learned in school (cf. De Bock et al., 2003). For example, students in the present study needed to see the relationship between the bills and coins and the school-learned algorithms for setting up and operating on decimal fractions. The presence of the perceptually rich bills and coins may have made this process more difficult because it is harder for children to use salient objects to represent abstract concepts (Uttal et al., 1997). When children's attention is drawn toward the physical quantities involved (e.g., the bills and coins), it may be pulled away from the abstract mathematical concepts being represented (DeLoache, Uttal, & Pierroutsakos, 1998; Uttal et al., 1997). Even when the perceptual details of an object seem to tap applicable real-world knowledge (e.g., knowledge of US currency), students may not automatically grasp the mathematical ideas when they look at the objects (cf. Ball, 1992; Gravemeijer, 2002; Meira, 1998). One dollar and one dime does not necessarily map transparently to $1.10.

In the present study, at least two aspects of the perceptually rich, realistic bills and coins may have drawn children's attention toward the objects themselves: (a) their inscriptions and (b) their resemblance to play money. The perceptually rich bills and coins were more colorful and more ornate than the bland ones. These perceptual details may have attracted students' attention to the bills and coins themselves and away from the decimal fractions being represented. At the same time, the perceptually rich bills and coins also may have activated children's knowledge of play money. Children often use bills and coins that resemble U.S. currency in their everyday play experiences (e.g., board games, pretend store play, etc.). Thus, they may already have a representation of the objects as toys, and it may be difficult for them to view the bills and coins as tools that can help them in school (cf. DeLoache, 1995; Moyer, 2001). Unfortunately, we cannot distinguish between these two alternatives with the current data; thus, an important goal for future research will be to tease apart the potential contributions of perceptual richness and activation of prior knowledge in children's difficulties with dual representation (McNeil & Jarvin, 2007).

The trade-offs observed in the present study may be used to support and extend cognitive load theory (Sweller, van Merriënboer, & Paas, 1998). According to cognitive load theory, educational contexts should be designed to

(a) maximize students' use of cognitive resources in activities that are relevant to the task at hand (germane load) and (b) minimize students' use of cognitive resources in activities that are not relevant to the task at hand (extraneous load). Previous research has shown a trade-off between these two goals in students with different levels of knowledge (the expertise reversal effect, Kalyuga et al., 2003). When students have low levels of knowledge, they need extra support from materials that foster germane load. However, when students have high levels of knowledge, extra support for germane load is redundant and hinders learning and performance by increasing extraneous load. In the present study, we saw a similar trade-off within a single group of students. The external support provided by the perceptually rich bills and coins seemed to benefit students' conceptualization of the word problems. At the same time, it also seemed to increase extraneous load and hinder students' execution of problem-solving procedures. This type of trade-off may only occur when problem solvers have intermediate levels of knowledge, as the fourth- through sixth-grade students had in the present study. Younger students who have not learned the school-based algorithms for operating on decimal fractions may not be able to solve the word problems without the help of bills and coins. In contrast, older students who have more domain knowledge may not reap any benefits from the presence of bills and coins. Future research should evaluate the effects of perceptually rich, realistic objects on students of different ages with different levels of mathematics experience.

4.2. Educational implications

How might teachers use the present findings to inform their practices in the classroom? We propose that teachers should try to maximize the advantages that concrete objects may provide while minimizing their disadvantages. Results suggest that using salient objects may be counterproductive when the goal is to reduce students' errors, as in a traditional testing situation. Thus, when teachers have this goal, they might consider not providing students with concrete objects, or providing students with concrete objects that are not perceptually rich or interesting in their own right. In this regard, there are many manipulatives (e.g., Base-10 blocks, Digi-blocks) that are relatively simple in nature. The simplicity of the form of the materials is a core concept in the design of Montessori curricula (Lillard, 2005).

Although they lead to more errors overall, perceptually rich, realistic objects may also convey some advantages for students' conceptualization of problems. Thus, when error-free performance is not the primary goal, teachers may want to provide students with such objects. For example, when students are working on problems through informal discussions, it may be beneficial for some errors to be made, so the class can identify and discuss them. Additionally, when teachers are providing instruction on a mathematical concept, perceptually rich objects may be helpful because they may activate real-world knowledge and help students think through contextually relevant interpretations.

However, even if perceptually rich, realistic objects do, indeed, foster children's informal understanding during discussion or instruction, there may still be reason for caution. It could be that the presence of such objects simply results in a greater divide between informal and formal knowledge. This outcome may be undesirable because much of mathematics involves written mathematical symbols. To succeed in mathematics and science, students need to understand what the formal symbols mean and how they can be manipulated. One of the primary goals of teachers should be to develop lessons that help students to bridge the gap between their informal knowledge of mathematics and their understandings of the corresponding formal symbolic representations (Greeno, 1989; Schoenfeld, 1988; Uttal, 2003). It may or may not be possible for teachers to use highly concrete objects to help students make such connections. The few studies that have touched on this issue suggest that perceptually rich concrete materials may discourage students from making such connections (Goldstone & Sakamoto, 2003; Sloutsky et al., 2005); however, it is still an open question because none of the studies to date have examined the effect of embedding concrete materials in math lessons organized around an authentic context. When problems are presented as modeling activities in authentic contexts, teachers may be able to highlight the connections between the concrete objects and the abstract concepts they represent.

4.3. Limitations and future directions

Although our results support the hypothesis that the presence of perceptually rich objects lead to trade-offs between informal understanding and execution of school-learned algorithms, several aspects of the current findings warrant additional research. Indeed, critics may argue that the increase in errors in the perceptually rich condition is just an artifact of the students' previous learning experiences. Students who participated in our study attended schools that use traditional curricula and teaching methods. They had been exposed to classroom money kits in informal ways in the classroom, but none had worked with the kits on a deep and consistent basis, as might be expected when mathematics lessons are presented as modeling activities in authentic contexts. Thus, the novelty of presenting the bills and coins during a test could have affected these students' motivation or beliefs about the task at hand in ways that increased errors.

For example, the presence of ''non-school'' objects during testing may have led students to be more easily distracted by extraneous features of the objects, which could have caused them to be more careless in their execution of solution strategies. It also may have led students to perceive the test as less difficult, which could have prompted them to try less than they would otherwise (De Bock et al., 2003). It also may have encouraged students to be overly concerned with figuring out the best way to use the objects to help them solve the problems. Students were not provided with prescribed ways of using the bills and coins, so they were left to invent their own methods. As a result, students who received bills and coins may have been confused, or may have been less motivated to carry out school-based algorithms, although, if this is the case, it is unclear why the students in the bland condition would be expected to perform differently under the same conditions. Nonetheless, the general point still holds. We should not generalize too far beyond the data. The trade-offs observed in the present study pertain to students' performance on word problems in a testing situation, and the effects may be limited to students who learn mathematics in traditional school settings. Future research should explore the effects of using different types of concrete objects in different types of learning and performance situations with students from traditional and nontraditional classrooms.

Finally, another limitation of the current study is that we chose to test students in the large classroom setting that is typical of most testing situations in the United States. Because of this choice, we were not able to collect the rich data on children's use of the bills and coins on a problem-by-problem basis that we would have been able to collect if we had tested children in one-on-one interviews. Without such data, we cannot provide specific details about how the presence of the bills and coins affected students' approach to various problems. Future studies should use a more process-oriented, richer methodology, so we can have a more fine-grained picture of the role of perceptually rich, realistic materials in students' problem-solving behaviors.

4.4. Conclusion

Taken together, the results of the present study highlight that the ultimate effects of activating real-world knowledge through the use of concrete objects may not be clear-cut. There may be both costs and benefits to providing students with perceptually rich, realistic objects to help them solve mathematics problems. Teachers and parents may want to weigh these costs and benefits when they are confronted with one of the many attractive toys being marketed as objects that facilitate children's mathematical problem-solving skills.

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