Gentner, D., & Gentner, D. R. (1983). Flowing waters or teeming crowds: Mental models of electricity. In D. Gentner & A. L. Stevens (Eds.), *Mental models* (pp. 99-129). Hillsdale, NJ: Lawrence Erlbaum Associates.

6 Flowing Waters or Teeming Crowds: Mental Models of Electricity

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Question: When you plug in a lamp and it lights up, how does it happen?
Subject Delta:... basically there is a pool of electricity that plug-in buys for you ... the electricity goes into the cord for the appliance, for the lamp and flows up *to-flows-I* think of it as flowing because of the negative to positive images I have, and also because . . . a cord is a narrow contained entity like a river.

Analogical comparisons with simple or familiar systems occur often in people's descriptions of complex systems, sometimes as explicit analogical models, and sometimes as implicit analogies, in which the person seems to borrow structure from the base domain without noticing it. Phrases like "current being routed along a conductor," or "stopping the flow" of electricity are examples.

In this paper we want to explore the conceptual role of analogy. When people discuss electricity (and other complex phenomena) in analogical terms, are they thinking in terms of analogies, or merely borrowing language from one domain as a convenient way of talking about another domain? If analogies are to be taken seriously as part of the apparatus used in scientific reasoning, it must be shown that they have real conceptual effects.

There are two lines of observational evidence (aside from the protocol cited) for the proposition that analogies can have genuine effects on a person's conception of a domain. First, analogies are often used in teaching, as in the following introduction to electricity (Koff, 1961).

> The idea that electricity flows as water does is a good analogy. Picture the wires as pipes carrying water (electrons). Your wall plug is a high-pressure source which you can tap simply by inserting a plug. . . . A valve (switch) is used to start or stop flow.

Thus, educators appear to believe that students can import conceptual relations and operations from one domain to another.

A more direct line of evidence is that working scientists report that they use analogy in theory development. The great astronomer Johannes Kepler wrote (quoted in Polya, 1973): "And I cherish more than anything else the Analogies, my most trustworthy masters. They know all the secrets of Nature, and they ought to be least neglected in Geometry [p. 12]." The Nobel Prize lecture of nuclear physicist Sheldon Glashow (1980) makes constant reference to the analogies used in developing the theory of the unified weak and electromagnetic interactions:

> I was lead to the group SU(2) x U(1) by analogy with the approximate isospin-hypercharge group which characterizes strong interactions... .
> Part of the motivation for introducing a fourth quark was based on our mistaken notions of hadron spectroscopy. But we also wished to enforce an analogy between the weak leptonic current and the weak hadronic current... .

These kinds of remarks are strongly suggestive of the conceptual reality of generative analogy. But people's understanding of their own mental processes is not always correct. It could be that, despite these introspections, the underlying thought processes proceed independently of analogy and that analogies merely provide a convenient terminology for the results of the process. This hypothesis, the Surface Terminology hypothesis, contrasts with the Generative Analogy hypothesis that analogies are used in generating inferences.

Our goal is to test the Generative Analogy hypothesis: that conceptual inferences in the target follow predictably from the use of a given base domain as an analogical model. To confirm this hypothesis, it must be shown that the inferences people make in a topic domain vary according to the analogies they use. Further, it must be shown that these effects cannot be attributed to shallow lexical associations; e.g., it is not enough to show that the person who speaks of electricity as "flowing" also uses related terms such as "capacity" or "pressure." Such usage could result from a generative analogy, but it could also occur under the Surface Terminology hypothesis.

The plan of this paper is to (1) set forth a theoretical framework for analogical processing, called structure-mapping; (2) use this framework to explore the analogies people use in the domain of electronic circuitry, based on evidence from introductory texts and from interviews; (3) present two experimental studies that test the Generative Analogy hypothesis; and finally, (4) discuss the implications of our findings for a general treatment of analogy in science.

A STRUCTURE-MAPPING THEORY OF ANALOGICAL THINKING

Just what type of information does an analogy convey? The prevailing psychological view rejects the notion that analogies are merely weak similarity statements, maintaining instead that analogy can be characterized more precisely (Miller, 1979; Ortony, 1979; Rumelhart & Abrahamson, 1973; Sternberg, 1977; Tourangeau & Sternberg, 1981; Verbrugge & McCarrell, 1977). We argue in this section that analogies select certain aspects of existing knowledge, and that this selected knowledge can be structurally characterized.

An analogy such as

1. The hydrogen atom is like the solar system.

clearly does not convey that *all* of one's knowledge about the solar system should be attributed to the atom. The inheritance of characteristics is only partial. This might suggest that an analogy is a kind of weak similarity statement, conveying that only some of the characteristics of the solar system apply to the hydrogen atom. But this characterization fails to capture the distinction between literal similarity and analogical relatedness. A comparable literal similarity statement is

2. There's a system in the Andromeda nebula that's like our solar system.

The literal similarity statement (2) conveys that the target object (The Andromeda system) is composed of a star and planets much like those of our solar system, and further, that those objects are arranged in similar spatial relationships and have roughly the same kind of orbital motion, attractive forces, relative masses, etc. as our system.

Like the literal comparison, the analogy (statement 1) conveys considerable overlap between the relative spatial locations, relative motions, internal forces, and relative masses of atom and solar system; but it does *not* convey that the objects in the two domains are similar. One could argue with the literal statement (2) by saying "But the star in the Andromeda system isn't yellow and hot." if the star happened to be a white dwarf. To argue with the analogical statement (1) by saying "But the nucleus of the atom isn't yellow and hot." would be to miss the point. The analogy, in short, conveys overlap in relations among objects, but no particular overlap in the characteristics of the objects themselves. The literal similarity statement conveys overlap both in relations among the objects and in the attributes of the individual objects.[1]

[1]An adequate discussion of literal similarity within this framework would require including a negative dependency on the number of nonshared features as well as the positive dependency on the number of shared features (Tversky, 1977). However, for our purposes, the key point is that, in

The analogical models used in science can be characterized as structure-mappings between complex systems. Such an analogy conveys that like relational systems hold within two different domains. The predicates of the base domain (the known domain)-particularly the relations that hold among the objects-can be applied in the target domain (the domain of inquiry). Thus, a structure-mapping analogy asserts that identical operations and relationships hold among nonidentical things. The relational structure is preserved, but not the objects.

In such a structure-mapping, both domains are viewed as systems of objects[2] and predicates. Among the predicates, we must distinguish between object attributes and relationships. In a propositional representation, the distinction can be made explicit in the predicate structure: *Attributes* are predicates taking one argument, and *relations* are predicates taking two or more arguments. For example, COLLIDE (x,y) is a relation, whereas RED (x) is an attribute. We will use a schema-theoretic representation of knowledge as a propositional network of nodes and predicates (cf. Miller, 1979; Rumelhart, 1979; Rumelhart & Norman, 1975; Rumelhart & Ortony, 1977; Schank & Abelson, 1977). The nodes represent concepts treated as wholes and the predicates express propositions about the nodes. The predicates may convey dynamic process information, constraint relations, and other kinds of knowledge (e.g. de Kleer & Sussman, 1978; Forbus, 1982; Rieger & Grinberg, 1977). Figure 6.1 shows the structure-mapping conveyed by the atom/solar system analogy. Starting with the known base domain of the solar system, the object nodes of the base domain (the sun and planets) are mapped onto object nodes (the nucleus and electrons) of the atom. Given this correspondence of nodes, the analogy conveys that the relationships that hold between the nodes in the solar system also hold between the nodes of the atom: for example, that there is a force attracting the peripheral objects to the central object; that the peripheral objects revolve around the central object; that the central object is more massive than the peripheral objects; and so on.

Structure-Mapping: Interpretation Rules

Assume that the hearer has a particular propositional representation of a known domain B (the *base* domain) in terms of object nodes b_t, b_2, . . . , b,, and predicates such as A, R, R'. Assume also a (perhaps less specified) representa-

analogy, a structural distinction must be made between different types of predicates. In Tversky's valuable characterization of literal similarity, the relation-attribute distinction is not utilized; all predicates are considered together, as "features." This suggests that literal similarity (at least in the initial stages of study) does not require as elaborate a computational semantics as metaphor and analogy.

2The "objects" in terms of which a person conceptualizes a system need not be concrete tangible objects; they may be simply relatively coherent, separable component parts of a complex object, or they may be idealized or even fictional objects. Moreover, often a target system can be parsed in

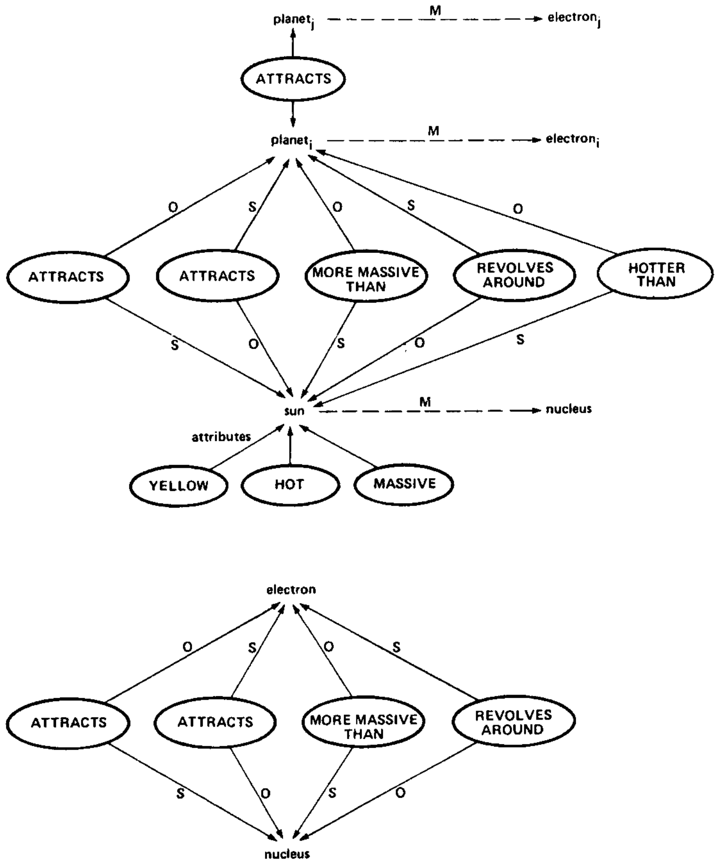


FIG. 6.1. Representations of knowledge about the solar system and the hydrogen atom, showing partial identity in the relational structure between the two domains.
Notes: S = subject; 0 = Object; M denotes mapping to a different system.

Lion of the domain of inquiry (the *target* domain) in terms of at least some object nodes t_t, t_2, . . . , t_m. Then a structure-mapping analogy maps the nodes of B into the nodes of T:

M: b_i -* t_i

The hearer derives analogical predications by applying predicates valid in the base domain B to the target domain T, using the node substitutions dictated by the mapping:

various ways by different individuals, or even by the same individual for different purposes. [See Greeno, Vesonder & Majetic (this volume) and Larkin (this volume).] The important point is, once the objects are determined they will be treated as objects in the mapping.

M: $[R(b_i,b_j)]$ -* $[R(t_i,t_i)]$

where $R(b_i,b_j)$ is a relation that holds in the base domain B. These analogical predications are subject to two implicit structural rules:

1. *Preservation of relationships. If* a relation exists in the base, then predicate the same relation between the corresponding objects in the target:

M: $[R(b_i, b)]$ -± $[R(t_i, t_i)]$

In contrast, attributes (one-place predicates) from B are not strongly predicated in T:

[A(bi)] -74 [A(ti)]

2. *Systematicity.* Sets of interconstraining relations are particularly important in explanatory analogy. Therefore, a relation that is dominated by a potentially valid higher-order relation is more strongly predicated than an isolated relation. For example, in the following expression, relations R, and R_2 are each dominated by the higher order relation R' that connects them. To the extent that any *of* these relations can be validly imported into the target, the strength *of* predication of the others is increased.

M: $[R'(R_1\ (b_i, b), R_2\ (b_k, b,))]$ =

[R'(R1 (ti, t), R2 (tk, tt))]

Preservation of Relationships. Assertion (1) states that relational predicates, and not object attributes, carry over in analogical mappings. This differentiates analogy from literal similarity, in which there is also strong attribute overlap. This follows from the central assertion that analogical mappings convey that identical propositional systems apply in two domains with dissimilar objects. For example, in the solar system model of the atom, the ATTRACTS relation and the REVOLVES AROUND relation between planet and sun are carried across to apply between electron and nucleus, whereas the separable *attributes* of the base objects, such as the color or temperature of the sun, are left behind. Mass provides a good illustration: The *relation* "MORE MASSIVE THAN" between sun's mass and planet's mass carries over, but not the absolute mass of the sun. We do not expect the nucleus to have a mass of 10^{30} kilograms, any more than we expect it to have a temperature of 25,000,000 °F.

Systematicily. Assertion (2) states that predicates are more likely to be imported into the target if they belong to a system of coherent, mutually constraining relationships, the others of which map into the target. These interconnections among predicates are explicitly structurally represented by higher-order relations

between those predicates (e.g., Smith, in preparation). One common higher-order relation is CAUSE; for example, CAUSE (R, , R_2) expresses a causal chain between the lower-order relations R, and R_2. Focusing on such causal chains can make an analogical matcher more powerful (Winston, 1981).

Figure 6.2 shows the set of systematically interconnected relations in the Rutherford model, a highly systematic analogy. Notice that the lower-order relations-DISTANCE (sun, planet), REVOLVES AROUND (planet, sun), etc.-form a connected system, together with the abstract relationship ATTRACTIVE FORCE (sun, planet). The relation MORE MASSIVE THAN (sun, planet) belongs to this system. In combination with other higher-order relations, it determines which object will revolve around the other. This is why MORE MASSIVE THAN is preserved while HOTTER THAN is not, even though the two relations are, by themselves, parallel comparisons. HOTTER THAN does not participate in this systematic set of interrelated predicates. Thus, to the extent that people recognize (however vaguely) that gravitational forces play a central role in the analogy they will tend to import MORE MASSIVE THAN, but not HOTTER THAN, into the target.

The systematicity rule aims to capture the intuition that explanatory analogies are about systems of interconnected relations. Sometimes these systems can be mathematically formalized. Some of the interrelations within this solar system are described in this equation:[3]

$$F_{grav} = Gmm'/R^2 \tag{1}$$

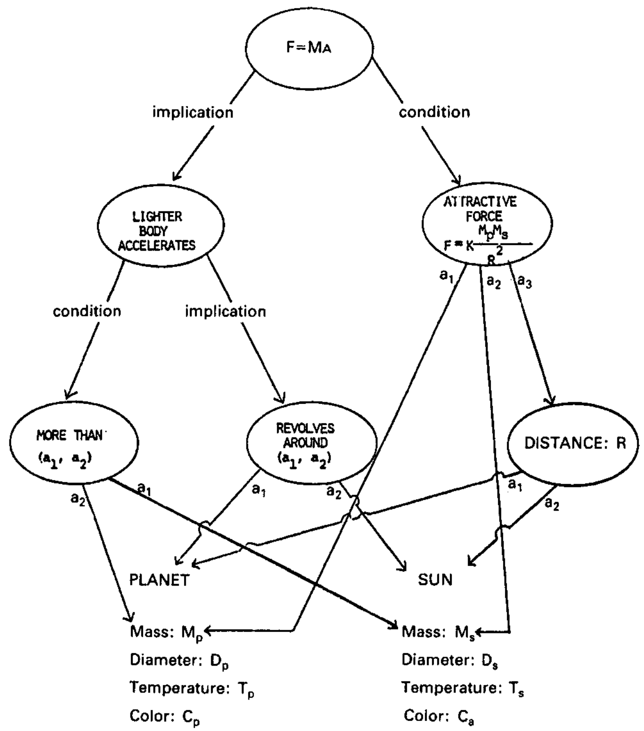
This equation embodies a set of simultaneous constraints on the parameters *of* the objects, where m is the mass *of* the sun, m' is the mass *of* the planet, G is the gravitational constant, and F_{grav} is the gravitational force. For example, *if* F_{grav} decreases while the masses are constant, then the distance R between the sun and the planet must increase. Equation (1) summarizing the interrelations in the base maps into a corresponding target equation:

$$F_{e,eC} = -qq'/R^2 \tag{2}$$

[3]Mathematical models represent an extreme of systematicity. The set of mappable relations is strongly constrained, and the rules for concatenating relationships are well-specified. Once we choose a given mathematical system-say, a ring or a group-as base, we know thereby which combinatorial rules and which higher-order relations apply in the base. This clarifies the process of deriving new predictions to test in the target. We know, for example, that if the base relations are addition (R,) and multiplication (R_2) in a field (e.g., the real numbers) then we can expect distributivity to hold: c(a+b) = ca + cb, or

$$R_2\ [(c, R, (a,b))] = R, [R_2\ (c,a), R_2\ (c,b)]$$

A mathematical model predicts a small number of relations which are well-specified enough and systematic enough to be concatenated into long chains of prediction.



(a)

FIG. 6.2. More detailed representation of knowledge about (a) the solar system and (b) the atom, ;;,showing partial identity in the higher-order relational structures between the two domains.

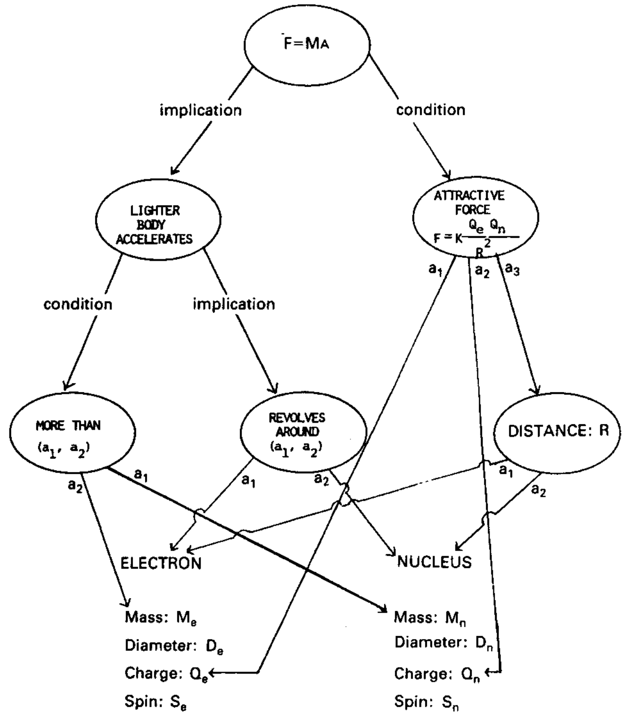
where q is the charge on the proton, q' the charge on the electron, R the distance between the two objects, and F_{etec} is the electromagnetic force.[4]

All these analogical predications are attempted predications, to use Ortony's (1979) term; they must be checked against the person's existing knowledge of the target domain. But the structural bias for relationality and systematicity provides an'iinplicit guide to which predications to check.

[4]Notice that the analogy shown in Fig. 6.2 actually involves two different systems of mappings that do not completely overlap. Each system is dominated by a different higher-order relation. Although the *object* mappings are the same in both cases, the *attribute* mappings are different. (Recall that object attributes, like objects themselves, can be mapped onto arbitrarily *different* elements of the target, according to the structure-mapping theory; only the resulting *relations* need be preserved.)

The first system of mappings is dominated by the attractive force relation

(F = G m, m_2 / R^2).



(b)

FIG. 6.2. (*continued*)

TWO ANALOGIES FOR ELECTRICITY

The domain of simple electricity is ideal for investigating the role of analogy. It is a familiar phenomenon; everyone in our society knows at least a little about it. Further, it is tractable: We can define ideal correct understanding. Yet because its mechanisms are essentially invisible, electricity is often explained by analogy. Moreover, because no single analogy has all the correct properties, we can compare different analogies for the same target domain. Finally, a great advan-

In this system, the *mass* of objects in the solar system is mapped onto the *charge of* objects in the atom. This system includes the higher-order relation that attractive force decreases with distance.

The other system is dominated by the inertial relation (F = ma); in this system, the *mass of* objects in the solar system maps into the *mass of* objects in the atom. This system includes the inference (expressed as a higher-order relation in Fig. 6.2) that the less massive object moves more than the more massive object.

tage of electronics is that, using simple combinations of circuit elements, it is easy to devise problems that require quantitative inferences that cannot be mimicked by mere lexical connections.

The Water-Flow Analogy

The analogy most frequently used to explain electricity is the water-flow analogy. We begin with this analogy, and later discuss an alternative analogy for electricity. The following passage is part of the instructions for a miniature lamp kit (Illinois Hobbycraft Inc., 1976).

ELECTRICITY AND WATER-AN ANALOGY

> An electrical system can be compared to a water system. Water flows through the pipes of a water system. Electricity can be considered as "flowing" through the wires of an electrical system.
> Wire is the pipe that electricity "flows" through. Volts is the term for electrical pressure. Milliamperes is the term for electrical "volume."

Here the base domain is a plumbing system and the object mappings are that a water pipe is mapped onto a wire, a pump or reservoir is mapped onto a battery, a narrow constriction is mapped onto a resistor, and flowing water is mapped onto electric current. What predicates is this analogy supposed to convey? Not that electricity shares object attributes with water, such as being wet, transparent, or cold to the touch. This analogy is meant to convey a system of relationships that can be imported from hydraulics to-electricity. In the next passages we discuss this relational structure, first for hydraulics and then for electricity. This will serve both to explicate the analogy and to provide some insight into electricity for readers who are unfamiliar with the domain. Then we compare the hydraulic analogy with another common analogy for electricity, the moving-crowd model.

Simple Hydraulics. We begin with a reservoir with an outlet at its base. The *pressure of* the water at the outlet is proportional to the height *of* water in the reservoir. (See Fig. 6.6, following.) The *rate of flow* through any point in the system is the amount of water that passes that point per unit time. Pressure and flow rate are clearly distinguishable: Rate *of* flow is *how much* water is flowing, while pressure is the *force* per unit area exerted by the water. Yet there is a strong relation between pressure and flow: The rate *of* flow through a section is proportional to the *pressure difference* through that section. This means that the greater the height *of* water in the reservoir, the greater the flow rate, all else being equal.

A constriction in the pipe leads to a drop in pressure. Water pressure, which is high when the water leaves the reservoir, drops across the constriction. Constrictions also affect flow rate: The greater the constriction in a section, the less the flow rate through that system. Figure 6.3b shows the relations among flow rate, pressure and degree *of* constriction for a hydraulics system.

The Analogy with Electricity. An electrical circuit is analogous to the plumbing system just described. Table 6. 1 shows the object correspondences, as well as some *of* the predicates that are imported from base to target. Notice that the predicates that are shared are relational predicates: for example, that increasing voltage causes an increase in current.

The first insight derivable from the analogy is the distinction between the flow rate and pressure, which maps onto an analogous distinction between current

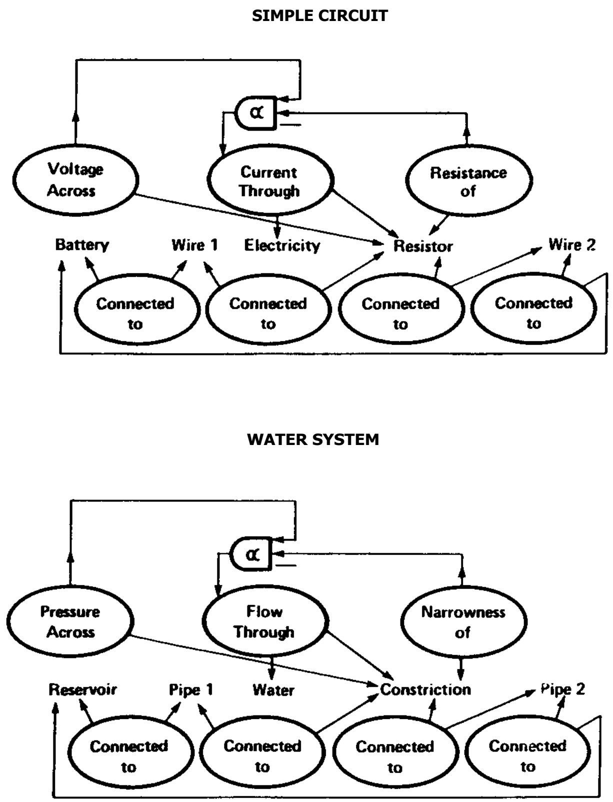


FIG. 6.3. Representation of knowledge about (a) simple electric circuits and (b) simple hydraulic systems, showing overlap in relational structures. The relation stands for a higher-order qualitative division relation: The output (e.g., current) varies monotonically with the positive input (e.g., voltage) and negative-monotonically with the negative input (e.g., resistance).

TABLE 6.1
Mappings Between Water Flow and Electricity

Base-Hydraulic System	*Target-Circuit*
Object Mappings:	
pipe	wire
pump	battery
narrow pipe	resistor
Property Mappings:	
PRESSURE of water	*VOLTAGE*
NARROWNESS of pipe	*RESISTANCE*
FLOW RATE of water	*CURRENT* (*FLOW RATE of* electricity)
Relations Imported:	
CONNECT (pipe, *pump*, narrow pipe)	*CONNECT* (wire, battery, resistor)
INCREASE WITH (*flow* rate, pressure)	*INCREASE* WITH (current, voltage)
DECREASE WITH (*flow* rate, narrowness)	*DECREASE* WITH (current, resistance)

(the number of electrons passing a given point per sec) and voltage (the pressure difference through which the current moves). This aspect of the analogy is important because novices in electricity often fail to differentiate current and voltage; they seem to merge the two of them into a kind of generalized-strength notion. For example, one subject, defining voltage, says:

> . . . Volts is . . . the strength of the current available to you in an outlet. And I don't know if it means there are more of those little electrons running around or if they're moving faster; ... .

Besides the current-voltage distinction, the analogy conveys the interrelation between current, voltage and resistance. Figure 6.3a shows the structural description of the circuit induced by the mapping. The batteries, wire, and resistors of an electrical circuit correspond to the reservoirs, pipes, and constriction of a plumbing system. Note the parallel interdependency relations in the two systems (Figs. 6.3a and 6.3b): e.g., Electrons flow through the circuit because of a voltage difference produced by the battery, just as water flows through the plumbing system because of a pressure difference produced by the reservoir. Thus, the analogy conveys the dependency relations that constitute Ohm's Law, = IR. Of course, naive users of the analogy may derive only simpler proportional relations such as "More force, more flow" and "More drag, less flow." These qualitative-proportion relationships (see Forbus, 1982) may be phenomenological primitives, in the sense discussed by diSessa (this volume).

The Moving-Crowd Model

Besides the hydraulics model, the most frequent spontaneous analogy for electricity is the moving-crowd analogy. In this analogy, electric current is seen as masses of objects racing through passageways, as in these passages from interviews:

> (1) You can always trick the little devils to go around or through... Because they have to do that. I mean, they are driven to seek out the opposite pole. In between their getting to their destination, you can trick them into going into different sorts of configurations, to make them work for you... .

> (2) If you increase resistance in the circuit, the current slows down. Now that's like a highway, cars on a highway where . . . as you close down a lane . . . the cars move slower through that narrow point.

The moving-crowd model can provide most of the relations required to understand electrical circuits. In this model current corresponds to the number of entities that pass a point per unit time. Voltage corresponds to how powerfully they push. Like the water analogy, the moving-crowd model establishes a distinction between current and voltage. Further, the moving-crowd model allows a superior treatment of resistors. In this model we can think of a resistor as analogous to a barrier containing a narrow gate. This "gate" conception of resistors is helpful in predicting how combinations of resistors will behave, as we describe in the following section. However, it is hard to find a useful realization of batteries in this model.

EXPERIMENTS ON ANALOGIES FOR ELECTRICITY

Rationale and Overview

The language used in the protocols suggests that people base their understanding of electronics at least in part on knowledge imported from well-known base domains. But are these true generative analogies or merely surface terminology? In order to verify that the use of a particular model leads to predictable inferences in the target domain, we performed two studies of analogical models in electronics. In Experiment 1, we elicited subjects' models of electronics and asked whether their models predict the types of inferences they make. In Experiment 2, we taught subjects different analogical models of electronics and compared their subsequent patterns of inference.

The Four Combinatorial Problems

We wished to test deep indirect inferences that could not be mimicked by surface associations. At the same time, we needed to keep our problems simple enough for novices to attempt. The solution was to ask about different combinations of simple components. There were four basic combination circuits, namely the four circuits generated by series and parallel combinations of pairs of batteries or resistors, as shown in Fig. 6.4. For example, we asked how the current in a simple circuit with one battery and resistor compares with that in a circuit with two resistors in series, or with two batteries in parallel.

The chief difficulty in these combination problems is differentiating between serial and parallel combinations. The serial combinations are straightforward: More batteries lead to more current and more resistors to less current. This accords with the first level of novice insight: the "More force, more flow/more

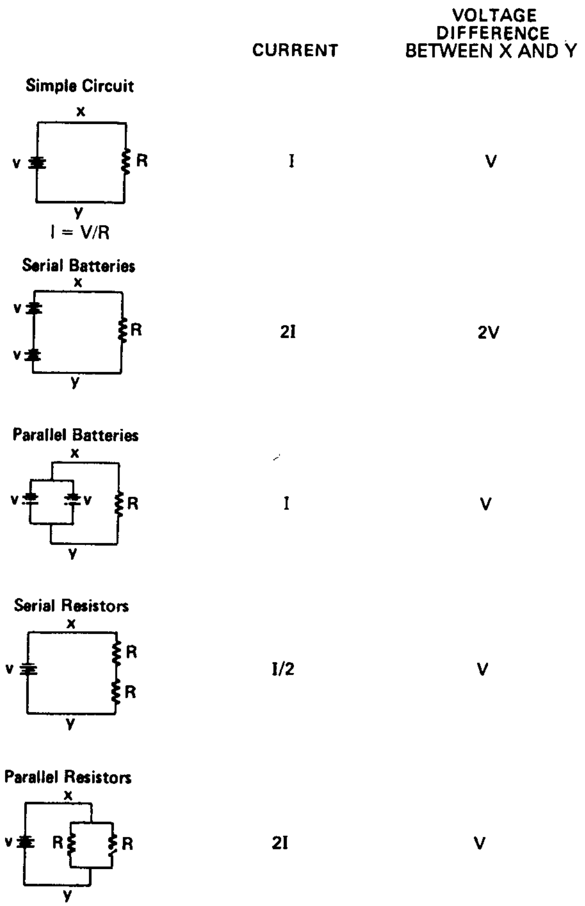


FIG. 6.4. Current and voltage for the four combination circuits: serial and parallel pairs of batteries or resistors. A simple battery-resistor circuit is shown at top.

drag, less flow." model, in which current goes up with the number of batteries and goes down with the number of resistors. But the parallel combinations do not fit this naive model: As Fig. 6.4 shows, parallel batteries give the *same* current as a single battery, and parallel resistors lead to *more* current than a single resistor (always assuming identical batteries and resistors).

Combinations of Batteries. To gain some intuition for these combinations, we return briefly to the water domain for a review of serial and parallel reservoirs. Consider what happens when two reservoirs are connected in series, one on top of the other. Because the pressure produced by the reservoirs is determined by the height of the water and the height has doubled, two reservoirs in series produce *twice* the original pressure, and thus *twice* the original flow rate. This conforms to the intuition that doubling the number of sources doubles the flow rate. However, if two reservoirs are connected in parallel, at the same level, the height of the water will be the same as with the single reservoir. Because pressure depends on *height,* not on total amount of water, the pressure and flow rate will be the same as that of the original one-reservoir system (although the capacity and longevity of the system will be greater).

Figure 6.5 shows the higher-order relationships comparing flow rate given parallel or serial reservoirs with flow rate in the simple one-reservoir system. The same higher-order relationships hold in the domain of electricity: The current in a circuit with two serial batteries is greater than current with a single battery. Current given two parallel batteries is equal to that given a single battery.

Combinations of Resistors. These combinations are understood most easily through the moving-crowd model, in which resistors can be thought of as gates. In the serial case, all the moving objects must pass through two gates, one after the other, so the rate of flow should be lower than for just one gate. In the parallel case, the flow splits and moves through two side-by-side gates. Since each gate passes the usual flow, the overall flow rate should be twice the rate for a single gate. Applying these relationships in the domain of electricity, [5] we conclude that serial resistors lead to less current than a single resistor; whereas parallel resistors lead to more current.

[5] In combinations of resistors, the key principle is that the voltage changes significantly only when current encounters a resistance. When the circuit contains two identical resistors in a row, the total voltage drop gets divided between the two resistors. Thus the voltage drop across each resistor is only half as great. As the current is proportional to the voltage drop, the current through each resistor is only half the original current. By conservation of charge, this reduced current is constant throughout the system. When the resistors are connected in parallel, each resistor has the full voltage drop across it. Therefore, current passes through each of the resistors at the original rate. This means that in the parts of the circuit where the two currents are united (before and after the resistors) the total current will thus be twice the current given one resistor.

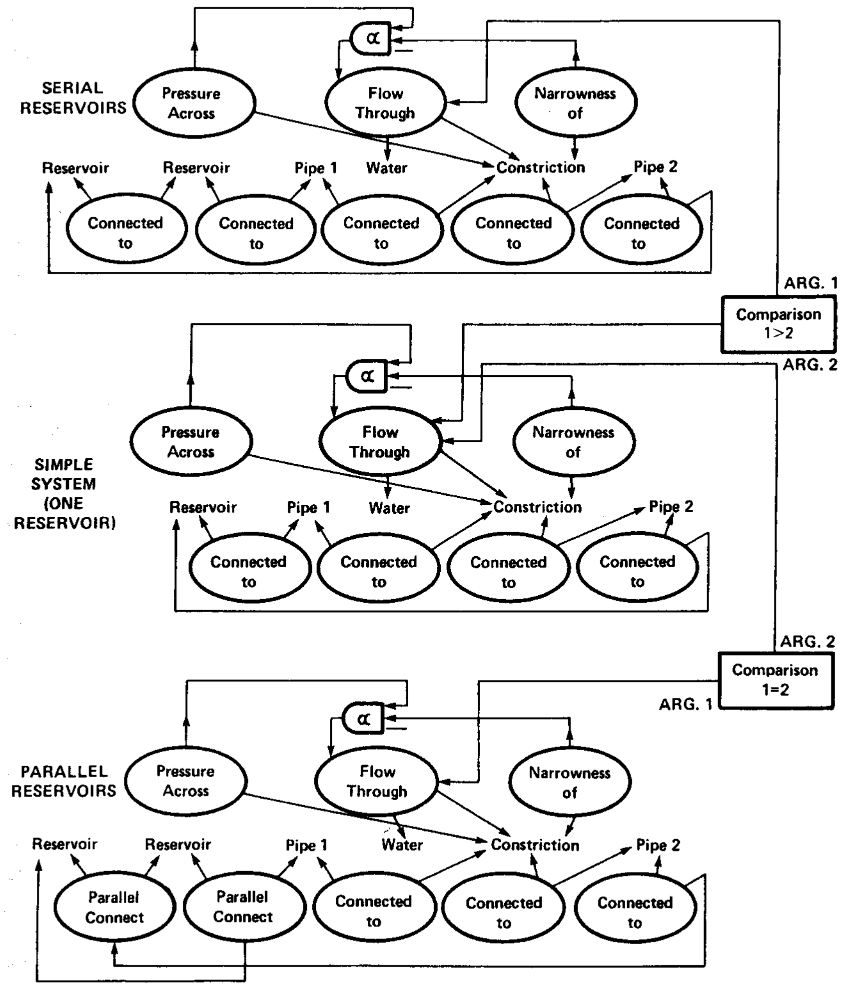


FIG. 6.5. Representation of knowledge in the hydraulic domain, showing higher-order comparison relations between rate of water flow in systems with parallel reservoirs and systems with serial reservoirs as compared with simple one-reservoir systems.

Predicted Differences in Patterns of Inference

The,flowing-water and moving-crowd models should lead to different patterns of performance on the four combination circuits. Both models can yield the first-stage "More force, more flow/more drag, less flow" law. Where the models should differ is in the ease with which further distinctions can be perceived. Subjects with the flowing-water model should be more likely to see the difference between the two kinds of battery combinations. Subjects with the moving-crowd model should be more likely to see the difference between the two kinds of resistor combinations.

Flowing fluid Model. Subjects who use the flowing-fluid model should do well on the battery questions. This is because, as described earlier, serial and parallel reservoirs combine in the same manner as serial and parallel batteries; thus already-familiar combinational distinctions can be imported from the water domain. However, subjects with the fluid flow model should do less well on resistor combinations. In the hydraulic model resistors are viewed as impediments. This often leads people to adopt the "More drag, less flow" view. Here, people focus on the idea that in both parallel and serial configurations the water is subjected to *two* obstacles rather than one. They conclude that two resistors lead to less current, regardless of the configuration.

Moving-crowd Model. For subjects with the moving-crowd model, the pattern should be quite different. In this model, configurations of batteries should be relatively difficult to differentiate, since it is hard to think of good analogs for batteries with the correct serial-parallel behavior. In contrast, resistors should be better understood, because they can be seen as gates. This should lead to better differentiation between the parallel and serial configurations, as described earlier. Subjects using this model should correctly respond that parallel resistors give more current than a single resistor; and serial resistors, less current.

The following protocol excerpt illustrates the superiority of the moving-crowd model for understanding parallel resistors. The subject began with the flowing-fluid model and incorrectly predicted less current in a parallel-resistor circuit:

> We started off as one pipe, but then we split into two.... We have a different current in the split-off section, and then we bring it back together. That's a whole different thing. That just functions as one big pipe of some obscure description. So you should not get as much current.

The experimenter then suggested that the subject try using a moving-crowd analogy. With this model, the subject rapidly derived the correct answer of *more* current for parallel resistors:

> Again I have all these people coming along here. I have this big area here where people are milling around.... I can model the two gate system by just putting the two gates right into the arena just like that.... There are two gates instead of one which seems to imply that twice as many people can get through. So that seems to imply that the resistance would be half as great as if there were only one gate for all those people.

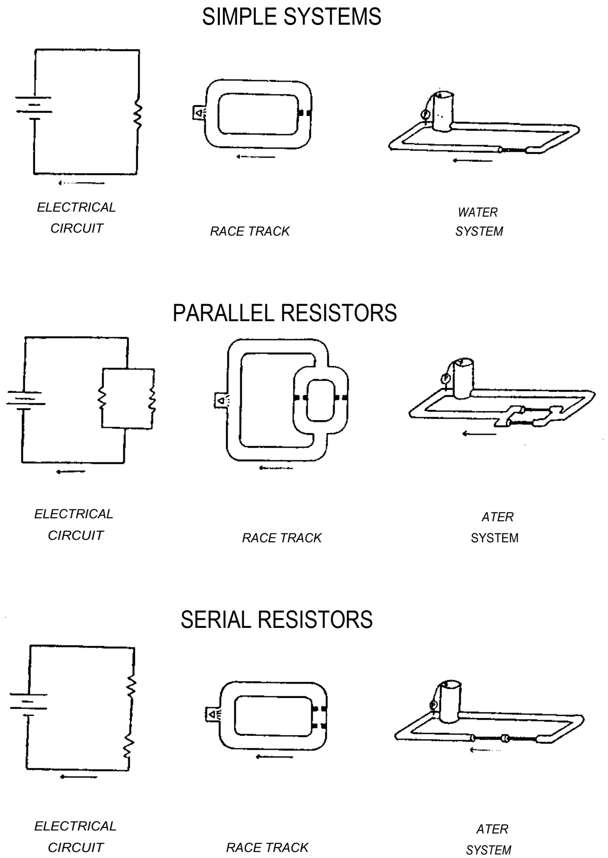


FIG. 6.6. Diagrams of electrical circuits, moving-crowd tracks and hydraulic systems, showing analogous systems for simple circuits, parallel-resistors circuits and serial-resistors circuits.

Figure 6.6 shows drawings of the analogs in the two systems, similar to those down by the subject. (Drawings of simple and serial-resistor systems are shown for comparison.)

These two sections of protocol suggest that models do affect inferences. The subject who drew incorrect conclusions using the water analogy later drew correct,.inferences using the moving-crowd analogy. The following study tests this pattern on a larger scale. If these models are truly generative analogies, we should find that the fluid-flow people do better with batteries than resistors, and the moving-crowd people do better with resistors than with batteries.

EXPERIMENT 1

Subjects

The subjects were 36 high school and college students, screened to be fairly naive about physical science. They were paid for their participation. Only subjects who used the same model throughout the study, as determined from their questionnaire responses, are included in the results discussed below. Also, among subjects who used a fluid-flow model, only those who correctly answered two later questions about the behavior of water systems were included. There were seven subjects who consistently used fluid flow models and eight subjects who consistently used moving-object models. The responses of subjects who were inconsistent in their use of models were analyzed separately and are not reported here.

Method

Qualitative Circuit Comparisons. Subjects were given booklets containing a series of questions and allowed to work at their own pace. The first page showed a simple circuit with a battery and a resistor, like the simple circuit in Fig. 6.4. Succeeding pages showed the four series-and-parallel combination circuits (see Fig. 6.4). They were asked to circle whether the current (and voltage) in each of the combination circuits would be greater than, equal to, or less than that of the simple battery-resistor circuit.

Questions About Models. After the subjects gave their answers for all four combination circuits, they were asked on a separate page to describe the way they thought about electricity, in their own words. On the next page, they were given a more specific choice: For each of the four circuit problems, they were asked to circle whether they had thought about flowing fluid, moving objects, or some other view of electricity while working on the problem. On the final page of the booklet they were asked questions about the behavior of reservoirs in the water domain.

Results

Figure 6.7 shows the results for subjects who reported using either the flowing-fluid analogy or the moving-crowd analogy consistently, on all four problems.

The patterns of inference are different depending on which model the subject had. As predicted, people who used the flowing-fluid model performed better on batteries than on resistors. The reverse is true for the moving-crowd people: they performed better with resistors, particularly in parallel, than with batteries. A

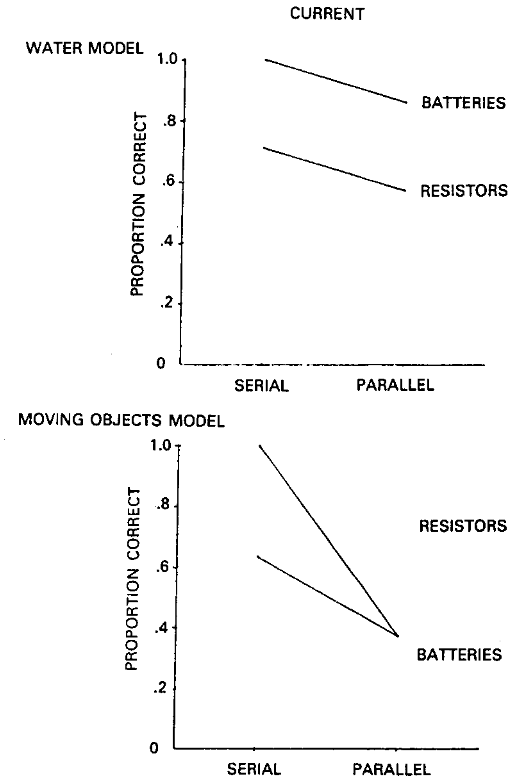


FIG. 6.7. Results of Experiment I: Proportions correct, for subjects with either a water-flow model or a moving-crowd model of electricity, on serial and parallel problems for batteries and resistors.

Model X Component X Topology 2 X 2 X 2 analysis of variance was performed on the proportions of correct answers. Here Model refers to whether the subject was using a flowing-fluid or moving-crowd model of electricity; Component refers to whether the combination was of batteries or resistors; and Topology refers to whether the problem involved a serial or parallel configuration. As predicted, the interaction between Model and Component was significant; $F(1,13) = 4.53$; $p < .05$. No other effects were significant.

Conclusions

The results of the study indicate that use of different analogies leads to systematic differences in the patterns of inferences in the target domain. Sdbjects with the flowing fluid model did better with batteries, while moving objects subjects did better with resistors. These combinatorial differences cannot be attributed to shallow verbal associations. These analogies seem to be truly generative for our

subjects; structural relations from the base domain are reflected in inferences in the target domain.

EXPERIMENT 2

In this study we taught subjects about electricity, varying the base domain used in the explanation. We then compared their responses to a series of questions about the target domain. Three different models of electronic circuitry were used. The first two models were versions of the hydraulic model, with fluid flow mapping onto current, pumps or reservoirs mapping onto batteries, pipes onto wires, and narrow pipes onto resistors. The two versions of this model varied according to what maps onto the battery: either a pump (Model P) or a reservoir (Model R). The third model was a moving-crowd model (Model M). In this model, current was seen as a moving crowd of mice and voltage was the forward pressure or pushiness of the mice.

The basic method was to present different groups of subjects with different models of electronics and then observe their responses to circuit problems. As in Experiment 1, the dependent measure is not merely percent correct but the pattern of responses. Each model should cause particular incorrect inferences as well as particular correct inferences. We also presented problems in the base domains. It seemed possible that subjects might have misconceptions in the base domains (such as hydraulics); in this case the knowledge available for importing into the target would deviate from the ideal knowledge.

Predicted Results

In the two hydraulics models, reservoirs (R) or pumps (P) are sources of pressure (voltage), which results in a flow of liquid (current) depending on the narrowness of the pipes (resistance). In the moving-crowd model, M, the forward pressure on the crowd (voltage), is generated by a loudspeaker shouting encouragement. This pressure creates a certain number of mice past a point per unit time (current) depending on the narrowness of the gates (resistance). Table 6.2 shows the correspondence among the three models.

Our major predictions were

1. that the moving-crowd model (M) would lead to better understanding of resistors, particularly the effects of parallel resistors on current, than the hydraulics models.

2. that the reservoir model (R) would lead to better understanding of combinations of batteries than either the moving-crowd model (M) or the pump model (P). With reservoirs, the correct inferences for series versus parallel can be derived by keeping track of the resulting height of water, as discussed earlier.

TABLE 6.2
Comparison of Water Flow, Moving Crowd, and Electricity Domains

Water Flow Models (R,P)	*Moving Crowd Model (M)*	*Electrical Circuit*
	Object Mappings	
hydraulic system	race course	circuit
water	mice	electricity or electrons
pipe	wide corridor	wire
pump or reservoir	loudspeaker	battery
constriction in pipe	gate in barrier	resistor
	Attribute Mappings	
PRESSURE OF water	PRESSURE OF mice	VOLTAGE
NARROWNESS OF pipe	NARROWNESS OF gate	RESISTANCE
FLOW OF pipe	PASSAGE RATE OF mice	CURRENT
	Relations between Objects that Hold in All Domains	
pump CONNECTED TO pipe	loudspeaker CONNECTED TO corridor	battery CONNECTED TO wire
pipe CONNECTED TO constriction	corridor CONNECTED TO gate	wire CONNECTED TO resistor
	Higher-order Relations that Hold in All Domains	
(FLOW OF water) *INCREASES WITH* (PRESSURE ACROSS constriction)	(PASSAGE RATE OF mice) *INCREASES WITH* (PRESSURE ACROSS gate)	(CURRENT) *INCREASES WITH* (VOLTAGE ACROSS resistor)
(FLOW OF water) *DECREASES WITH* (NARROWNESS OF constriction)	(PASSAGE RATE OF mice) *DECREASES WITH* (NARROWNESS OF gate)	(CURRENT) *DECREASES WITH* (RESISTANCE)
[SUM OF (FLOW INTO point)] *EQUALS* [SUM OF (FLOW OUT OF point)]	[SUM OF (PASSAGE RATE INTO POINT)] *EQUALS* [SUM OF (PASSAGE RATE OUT OF point)]	[SUM OF (CURRENT INTO point)] *EQUALS* [SUM OF (CURRENT OUT OF point)]
	Relations that Do Not Hold in All Domains	
RATIO OF (cross-section WIDTHS OF pipe and constrictions) is typically 100:1	*RATIO OF* (WIDTHS OF corridor and gate) is typically 10:1	*RATIO OF* (RESISTANCES OF resistor and wire) is typically 1,000,000:1

Neither the pump analog nor the loudspeaker analog has as clear a combination pattern.

Method

Subjects. Eighteen people participated, all either advanced high school or beginning college students from the Boston area. Subjects had little or no prev~ous knowledge of electronics. They were paid for their participation. Due to experimenter's error, there were seven subjects in the M group, six in the P group and five in the R group.

Procedure. After filling out a questionnaire concerning their general backgrounds, subjects were divided into three groups, each receiving different models. The procedure was as follows:

1. *Model-teaching.* Subjects were given a brief introduction to electricity consisting of Ohm's Law (I=V/R) together with an explanation of one of the three models.
2. *Simple test.* All three groups were given an identical set of five simple circuit problems to calculate. In each case the circuit was a simple battery-plus resistor circuit, and subjects solved for current, voltage or resistance by applying Ohm's Law. We required that subjects solve at least four problems correctly to be included in the study.
3. *Qualitative comparisons.* Subjects were next shown diagrams of the four complex circuits (SB, PB, SR, and PR, as shown in Fig. 6.4) along with a diagram of a simple battery-resistor circuits. For each such complex circuit, we asked subjects to compare current and voltage at several points in the circuit with that of the corresponding point in a simple circuit; e.g., they were asked whether current just before the resistors in a parallel-resistor circuit is *greater than, equal to* or *less than* the corresponding current in a simple circuit.
4. *Quantitative scaling.* Each subject received each of the four kinds of complex circuits (SB, SR, PB or PR) and filled out a series of scales indicating current and voltage at the same test points as in task (3).
5. *Drawing base given target analog.* Each subject received, for each of the four complex circuits, a sheet containing a simple base version of the standard simple system (analog of battery plus resistor); and a circuit drawing of one of the four complex circuits (SB, SR, PB or PR). They were told to draw the *base* version of the complex circuit shown.
6. *Base qualitative questions. To* test knowledge of the base system, subjects were given a picture of one of the four complex systems in the base, and answered qualitative questions about pressure and flow rate in the base system. Each sheet showed a simple system (the analog of battery plus resistor) plus a complex system (the analog of SB, SR, PB or PR). The subjects made judgments at the same points as in tasks *(3)* and (4).
7. *Thought questions.* Subjects were asked to write out answers to questions such as "What will happen if there is no resistor in the circuit?"; and "Do electrons go faster, slower or the same speed through the resistor as through the wire?"

Results: Prediction 1

Results supported the first prediction, that the moving-crowd model (M) would lead to better performance on parallel-resistor problems than the water models (P and R).

TABLE 6.3
Results of Experiment 2:
Performance on Problems Involving
Current with Parallel Resistors

	M	P	R
Qualitative Comparisons[a]	.93	.58	.70
Quantitative Scaling[b]	.71	.50	.40

[a] Proportions of responses that current in parallel-resistor circuit is greater than or equal to current in simple one-resistor circuit,

[b] Proportions of responses that current in parallel-resistor circuit is greater than current in simple circuit.

Qualitative Comparisons. In the M group, 93% of the subjects answered that current given two parallel resistors would be greater than or equal to current given a single resistor, as compared with .63 for the combined P and R groups. This difference between the M group and the P and R groups combined was significant by a X^2 test ($p < .05$). Table 6.3 shows the results for current given parallel resistors both for the qualitative comparisons task and for the quantitative scaling task.

The pattern of M-superiority on parallel-resistor problems also obtained for voltage. The proportions of questions in which subjects (correctly) answered that the voltage in a circuit with two parallel resistors is equal to the voltage in the simple circuit with one resistor were, for the M group, .86; for the P group, .42; and for the R group, .50. Again, the M group is significantly different from the combined P and R groups by a X^2 test ($p < .025$); M differs from P significantly as well (Fisher test, $p < .05$).

Quantitative Scaling. The differences, though nonsignificant, were in the predicted direction, as shown in Table 6.3. The proportions of times subjects answered that current in a parallel-resistor circuit would exceed current given a single resistor were .71 for M, .50 for P, and .40 for R. For voltage, the proportions of times subjects answered that voltage in a parallel-resistor circuit equals that in a simple circuit were .86 for M, .83 for P and .60 for R.

Results: Prediction 2

Our second prediction, that the R group would be superior to the M and P groups ons parallel-batteries problems, was not supported.

Qualitative Comparisons. The proportions of times subjects correctly answered that the voltage given parallel batteries is equal to the voltage given a single battery were .40 for the R group, .64 for the M group, and .33 for the P group. None of these differences was statistically significant.

For serial-battery problems, we expected less difference between the groups. This is because the correct answer-that voltage is greater in a circuit with two batteries in serial than with just one battery-is derivable from several different models, even from the naive "More force, more *flow"* view. The results are that the proportion of correct responses was .60 for R and .50 for P; for the M group, it was .57 (no significant differences).

Quantitative Scaling. Again we failed to find clear evidence that the R group understood parallel-battery problems better than the P group. The proportions of correct answers (that voltage is the same for PB as for a simple circuit) were .2 for R and .33 for P. The R group did perform better on the serial battery problems: .8 of the R answers indicated more voltage with serial batteries, whereas only .33 of the P answers did so. None of these differences is significant. (This lack of significance may seem surprising; however, we had only one data point per subject.) Rather surprisingly, the M group, with .86 correct, was significantly better than the other two groups on parallel batteries ($p < .025$, X^2).

Other Results in the Qualitative Comparison and Quantitative Scaling Tasks

There were two other significant differences. First, in the qualitative comparisons task, the P group was superior to the R group for current in a serial-resistor circuit. The proportion of times subjects correctly answered that current is lower with two serial resistors than with a single resistor was .58 for P and .10 for R ($p < .05$). There were no other significant differences on the qualitative comparison task.

The other remaining significant result is that, in the quantitative scaling problems, the R group performed better (at .40 correct) than the M group (0 correct) or P group (0 correct) on answering that current is constant everywhere in a purely serial circuit (such as SB or SR). The difference between R and P is significant ($p < .05$) as well as the difference between R and M ($p < .025$). This issue of constant steady-state current *flow* seems quite difficult for subjects, as discussed next.

Subjects' Knowledge of the Base. We were puzzled by the failure of Prediction 2: the finding that the R group did not excel at combinations of batteries, in spite of the seeming transparency of the corresponding combinations in the reservoir domain. One possible explanation is that, contrary to our intuitions, our subjects did not understand serial and parallel reservoirs any better than they understood serial and parallel pumps or loudspeakers. To check this possibility, we examined the subjects' answers in the base domains.

The results of the Base Qualitative Comparisons task revealed that subjects indeed failed to grasp the distinction between parallel and serial pressure sources in the base domains. Scores on the qualitative comparison problems concerning rate of flow of water or animals (analogous to current) were .35 for R, .42 for P and .32 for M. It is not surprising, then, that the R subjects failed to make correct inferences in the target domain of electricity.

Subsequent interviews have borne out the suspicion that even college-educated people fail to understand the way water behaves. They have difficulties not only with series versus parallel combinations of reservoirs or pumps, but also with the notion of steady-state flow. Current is seen not as a steady flow, constant throughout the system, but rather as a progression: Flow is strong and rapid at the source and gradually weakens as it goes through the pipes, with a drastic cut-back as it goes through the constriction. Moreover, people often fail to make the distinction between flow rate and related physical variables. Many people seem to have *a generalized strength-attribute* which is a composite of velocity, pressure, force of water, and rate of flow. This strength is thought to be very high at the outset, just after the reservoir, to diminish as the water travels around the water system, and to decrease sharply at the constriction.

Similar misconceptions show up in electronics. People in interviews do appear to have a kind of composite strength attribute that is interchangeably referred to as current, voltage, velocity of the electrons, power, pressure, or force of the electrons. This strength attribute fails to obey steady-state: It decreases as the stuff flows around the circuit, with the sharpest diminution occurring at the resistor.

The subjects' misconceptions in electronics are strikingly analogous to those in hydraulics. Therefore, subjects' failure to import veridical differentiations from the base domain does not constitute evidence against the Generative Analogies hypothesis. Even a fully generative, rigorous structure-mapping process cannot produce correct distinctions 'in the target domain unless subjects have grasped these differentiations in the base domain. Our investigations bring home the point that an analogy is only useful to the extent that the desired relational structure is present in the person's representation of the base domain.

DISCUSSION

It is an appealing notion that analogies function as tools of thought (Clement, 1981; Darden, 1980; Dreistadt, 1968; Hesse, 1966; Hoffman, 1980; Jones, in preparation; Oppenheimer, 1955). In this research we have sought to bring psychological evidence to bear on this claim.

We first noted that we find analogical references in people's spontaneous discussions of natural phenomena; for example, when a person discusses electric current in terms of traffic or in terms of flow of water. Our protocols suggest that people use analogies to help structure unfamiliar domains. The pervasiveness and generative quality of people's analogical language suggests that the analogies are used in thinking (Lakoff & Johnson, 1980; Quinn, 1981; Reddy, 1979; Schon, 1979). But to make this conclusion it must be demonstrated that the thinking truly depends on the analogy: that the analogy is more than a convenient vocabulary in which to discuss the results of independent inferential processes.

Evidence for the conceptual role of analogy comes from the introspections of creative scientists. The journals and self-descriptions of scientists from Johannes Kepler (1969; see also Koestler, 1963) to Sheldon Glashow (1980) seem to lean heavily on analogical comparisons in discovering scientific laws. Glashow's account of his use of generative analogies in nuclear physics was quoted earlier. Kepler's journals show several signs of generative analogy use. First, he makes reference to the analogy in stating his theory. Second, he appears to derive further insights from the analogy over time. Finally, as quoted earlier in this chapter, Kepler himself states that he uses analogy to further his thinking. The tempting conclusion is that, for scientists like Kepler and Glashow, analogies are genuine conceptual tools.

However, self-reports concerning psychological processes are not conclusive evidence, as Nisbett and Wilson (1977) have argued. In this research we tested the Generative Analogy hypothesis that analogy is an important source of insight by asking whether truly different inferences in a given target domain are engendered by different analogies. We chose as our target domain simple electricity, partly because it has the right degree of familiarity, and partly because there are two good, readily available base domains-flowing water and moving crowds-that support different inferences in the target domain.

To test this hypothesis, we needed to find problems for which the inferences required in the target could not be mimicked by verbal patterns, but would reflect structural relations imported from these different base domains. We chose the four combinatorial problems described earlier: serial and parallel combinations of resistors and batteries. These problems are simple enough to be posed even to a novice, yet are nontransparent enough that they require some sustained thought. We predicted that the parallel-serial distinction for batteries should be clearer using flowing fluid as the base. This is because the pressure difference between serial and parallel reservoirs can be understood in terms of height of fluid, a relatively accessible distinction. Therefore, use of the water system as a base domain should improve understanding of batteries. In contrast, the parallel-serial distinction for resistors should be more obvious using the moving-crowd base domain. In the moving-crowd model, resistors can be thought of as gates (inferior passages) rather than as obstructions. Subjects who use that model should see that parallel resistors, analogous to gates side by side, will allow more flow than a single resistor. The opportunity is there to find effects of thinking in different analogical models.

In Experiment 1, we divided subjects according to which analogy they reported using for electricity and compared their inferences about the current in our four combination problems. We found, as predicted, that subjects using the water model (given that they understood the way water behaves) differentiated batteries more correctly than resistors, and that subjects who used the moving-crowd model were more accurate for resistors than for batteries. These results support the generative analogies claim of a true conceptual role for analogical models. The pattern of inferences a subject made in the target domain did indeed match the pattern that should have been imported from the base domain.

Experiment I provided evidence for the Generative Analogies hypothesis for people's preexisting spontaneous analogies. Experiment 2 examined the effects of analogical models that were taught to subjects. In Experiment 2, we taught people to use one of three models and compared their subsequent patterns of inference. If people's inferential patterns varied according to the model they were taught, this would provide a second line of evidence for analogical reasoning. We found some of the predicted effects in Experiment 2. Subjects who were taught the moving-crowd analogy could differentiate parallel versus serial resistor configurations more accurately than subjects who had learned either of the water models. However, we did not find the predicted differences in ability to differentiate the two types of battery combinations.

We suspect that there are two main reasons that the results of Experiment 2 were weaker than those of Experiment 1. The first problem was that we did not screen people for knowledge of the water domain in Experiment 2. In many cases, people simply did not understand that serial reservoirs and parallel reservoirs yield different pressure in the domain of water. Because we had information concerning subjects' knowledge of the respective base domains, we were able to demonstrate that in many cases the failure of the analogical inference was due to the lack of the corresponding inference in the original base domain.

The phenomenon of mapping erroneous knowledge may be fairly widespread. Several independent researchers have reported that mental representations of physical phenomena-even among college populations-often contain profound errors. Yet, although these initial models may be fragmentary, inaccurate, and even internally inconsistent, nonetheless they strongly affect a person's construal of new information in the domain (Brown & Burton, 1975; Brown, Collins & Harris, 1978; Chi, Feltovich, & Glaser, 1981; Clement, 1981, this volume; diSessa, this volume; Eylon & Reif, 1979; Gentner, 1980, 1982; Hayes, 1978; Hollan, Williams & Stevens, this volume; Larkin, this volume; McCloskey, this volume; Miyake, 1981; Sayeki, 1981; Stevens & Collins, 1980; Stevens, Collins &,,Goldin, 1979; Wiser & Carey, this volume). Our research, and that of other investigators, suggests that these domain models, whether correct or incorrect, are carried over in analogical inferencing in other domains (Collins & Gentner, in preparation; Darden, 1980; Gentner, 1979; Johnson-Laird, 1980; Riley, 1981; VanLehn & Brown, 1980; Winston, 1978, 1980, 1981; Wiser & Carey, this volume).

Aside from the subjects' lack of insight in the base domain, the second problem with Experiment 2 is that the teaching sessions may have been inadequate to convince all the subjects to use the models. People simply read a one-page description of the model that they were to learn, and then began answering questions. Accepting a new model often requires considerable time and practice. The problem of convincing subjects to use a particular model did not exist in Experiment 1; subjects were sorted according to the model they reported using a priori. This possible pattern of conservatism in use of new models accords with that found in experimental studies of analogical transfer by Gick and Holyoak (1980), and Schustack and Anderson (1979). Both these studies found that, although subjects are demonstrably able to import relational structure from one domain to another, they often fail to notice and use a potential analogy. We suspect that one reason subjects may be slow to begin using a new analogy for an area is that they normally enter a study with existing models of the domain.

However, although Experiment 1 produced stronger results than Experiment 2, the results of the two experiments taken together provide clear evidence for the Generative Analogies hypothesis. People who think of electricity as though it were water import significant physical relationships from the domain of flowing fluids when they reason about electricity; and similarly for people who think of electricity in terms of crowds of moving objects. Generative analogies can indeed serve as inferential frameworks.

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