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CHAPTER

David H. Uttal
Linda L. Liu
Judy S. DeLoache

Taking a Hard Look at Concreteness: Do Concrete Objects Help Young Children Learn Symbolic Relations?

Introduction

The abiiity to understand and manipulate symbols is one of the most important milestones of human development. Symbols allow us to contemplate ideas and concepts after they cease to be directly perceived. Symbol systems such as language provide the means for conveying this information to others, allowing learning to occur independent of direct perception or experience. Symbols such as numbers also serve a notational role, obviating the need to manipulate physical quantities, making it possible to perform operations on abstract representations of the quantities.

Given the importance of symbols, it is not surprising that considerable research has been devoted to improving children's understanding of symbols. Much of the research on symbolic development shares a common assumption with other research on children's thinking—the idea that the thinking of young children is very concrete. Developmental psychologists typically have characterized younger children as thinking in the "here and now" and older children as being capable of reasoning beyond their direct experience (Bruner, 1966; Piaget, 1951; Vygotsky, 1934/1962; Werner & Kaplan, 1963). Young children are seen to rely heavily on tangible, real-world information while older children possess general concepts that integrate the essence of these concrete experiences. In sum, the idea that concrete thinking precedes abstract thought lies at the heart of many prominent theories of development.

The assumption that young children's thinking focuses on concrete properties has led to further assumptions about children's understanding of symbols. Specifically, from the belief that children attain understanding at a concrete level first, it has been inferred that the most appropriate means for teaching them about symbols is to make the symbols palpable and perceptually salient (Ball, 1992; Montessori, 1917). Using concrete objects for instruction is supposed to train children to think about objects apart from their physical reality and to lay the foundation for the ability to use and understand abstract symbols. Bruner (1966) described this training process as learning to "empty the concept of specific sensory properties [in order to] grasp its abstract properties" (p. 65).

In this chapter, we argue for a different perspective on the role of concreteness in children's thinking. Reducing development to a concrete-to-abstract shift may both oversimplify the nature of children's early thinking and overestimate the extent to which objects are appropriate for the instruction of young children. We question the view that concrete materials necessarily improve children's understanding of symbols and abstract concepts. Furthermore, we suggest that concreteness is a two-edged sword: It can help young children to detect symbolic relations but also can make it more difficult for them to comprehend the abstract concepts represented by the symbolic objects. In support of this claim, we review research on children's understanding of symbolic relations in several domains. We begin, however, with a brief discussion of the traditional theoretical basis for assuming that concreteness is of paramount importance in the development of young children's understanding of symbolic relations. We also review the work of some theorists who have challenged this assumption.

Symbols and General Cognitive Development

The idea that children understand symbols best through concrete objects is derived, in part, from the work of many classic developmental theorists. The acquisition of symbolic competence is seen to proceed through a concrete-to-abstract shift: the progression from thinking that is rooted in concrete reality to thinking that is unconstrained by context. Sigel (1993) described this developmental progression as the child's attempt to "separate him- or herself mentally from the ongoing here and now, and project him- or herself to some other temporal plane (past or future or the nonpalpable present), in turn transforming the received communication into some symbol or sign system" (p. 142). The end result of this progression is that the concepts of older children are no longer direct, iconic representations of their original encounter with specific instances of an object; they are reconceptualizations or abstractions based on the original stimulus. Consequently, development often has been characterized as children's struggle to transcend their shallow and shortsighted view of the world (Bruner, 1966; Piaget, 1951; Werner & Kaplan, 1963).

The idea that young children's thinking is based on concrete concepts runs through almost all classic theories of cognitive development more generally and symbolic development more specifically. For example, Inhelder and Piaget (195511958) considered the ability to use abstract, hypothetical information, without reference to more concrete information, to be a hallmark of the qualitative shift to formal-operational reasoning. The ability to perform logical deductive reasoning from false propositions (i.e., independent of reality) is one test of the type of abstract thought characteristic of Piaget's stage of formal operations. Given a set of statements such as "If mice are bigger than dogs and dogs are bigger than elephants," young children typically have trouble making the correct deduction "then mice are bigger than elephants." Since none of these relations exist in reality, young children are left without a concrete basis from which to reason and solve the problem (Moshman & Franks, 1986).

Indeed, it is children's attempt to rely on familiar, real relations among entities that makes counterfactual reasoning especially difficult for them. Thinking of a real mouse and a real dog, they have trouble mentally reversing the usual size relations among them. Consistent with this, Dias and Harris (1988, 1990) have shown that young children perform better when counterfactual reasoning tasks are presented in a make-believe rather than a literal mode. For example, they presented children with pairs of premises that clearly violated the children's real-world knowledge such as "All cats bark" and "Rex is a cat." Dias and Harris found that children were more likely to correctly answer "yes" to the question, "Does Rex bark?" when they were first told to pretend they were on another planet or when the experimenter presented the premises using "make-believe intonation." Thus, in some situations, the concreteness of children's real-world knowledge presents an obstacle to their ability to perform abstract reasoning.

Other theorists have discussed the concrete-to-abstract shift under various guises. In studies of early categorization, Bruner, Goodnow, and Austin (1956) described conceptual development as a perceptual-to-conceptual shift; children first think of objects only in terms of the properties directly available to their senses but can later begin to consider their abstract properties. Children may, for example, initially categorize birds and bats together because they share the perceptually salient feature of being flying creatures. As children become aware of the abstract basis of the categories, however, they not only realize that birds and bats belong in separate categories but that flightless penguins belong in the bird category. Thus, according to this view, children learn to rely less on concrete physical attributes for categorization and more on the underlying structure of the categories.

Similarly, Vygotsky (193411962) extended the idea of concreteness through two lines of study. First, Vygotsky introduced the notion of situational thinking, proposing that children demonstrate a thematic-to-taxonomic shift in how they classify objects. Thematic categories (e.g., bee, hive, honey) capture the fleeting but salient relations between objects in a common setting rather than the deeper structure that defines the systematic relations among objects of like kind. Thus, the developmentally primitive categories are, in essence, highly concrete because they are rooted in a specific situation that is salient to the child. Progression to taxonomic categorization (e.g., bee, fly, grasshopper) requires that the child reject the perceptually salient features of the objects in favor of their deeper similarities.

In a second line of study, Vygotsky proposed that children learn to consider the deeper similarities between objects through make-believe play. He noted that, when children play make-believe, they often substitute concrete objects for objects in the real world (e.g., a stick for a horse). He theorized that these concrete objects essentially serve as symbols because the game of make-believe strips them of their typical identity. Consequently, children become capable of seeing the physical object as being separable from its abstract meaning or identity.

The concrete-to-abstract idea also has been applied to the development and understanding of symbols. Werner and Kaplan (1963) conceptualized development as a holistic-to-analytic shift in which young children first attend to "physicochemical stimuli" from the environment which are later converted into "stimulus-signs or signals" by the developing child (p. 9). For example, young children presented with the number "3" might attend to the concrete properties of the stimulus (e.g., its curvature and symmetry). On the other hand, older children more likely would be capable of analyzing or focusing on the deeper, abstract symbolic meaning of the number.

Implications of the Concrete-to-Abstract Shift

Theorists proposing a concrete-to-abstract shift have not necessarily advocated an inflexible, dichotomous view of early versus later development. Unfortunately, the oversimplified assumption often has been made that younger children benefit from, and indeed require, concrete instruction. Ball (1992) characterized this assumption as "Concrete is inherently good; abstract is inherently not appropriate—at least at the beginning for young learners" (p. 16). Concreteness thus has come to be regarded as a panacea for improving young children's learning of abstract symbols and concepts; educators often have assumed that abstract or symbolic information can best be communicated by making it concrete.

Researchers recently have become wary of taking the educational value of concreteness for granted. The notion of an inflexible concrete-to-abstract shift increasingly has been challenged as evidence accumulates showing that younger children do not rely on or benefit from concreteness as much as previously thought (Simons & Keil, 1995; Uttal, Scudder, & DeLoache, 1997). Concrete-to-abstract theories typically presume not only that concrete thought precedes abstract thought, but that it precludes it.

Simons and Keil (1995) went so far as to suggest that children's development of biological thought may in fact prcoceed in an abstract-to-concrete shift. They argued that children may first learn to make causual explanations for events at an abstract level because they lack specific physical knowledge about the events. For example, a child explaining the function of a camera might describe its ability to capture "brief temporal slices of reflected light patterns," but might not provide a mechanistic account detailing how light enters the lens and how the various parts of the camera interact (p. 131). In other words, children's earliest causal explanations may be general and abstract rather than concrete because they lack the domain-specific knowledge required to describe the specific components of the camera. Simons and Keil (1995) summarized their viewpoint stating that, "Although ignorance of the physical components of a system may preclude a concrete explanation for the system's behavior, it is quite possible to generate a principled, abstract explanation without any knowledge of the physical components" (p. 131). Thus, they emphasized that, although young children's explanations are abstract, they are not ignorant.

Similar arguments for young children's ability to think in abstract terms were presented by Gelman and Wellman (1991). They suggested that children understand that certain objects have an internal "essence" that is distinct from the outward appearance of the objects. Like Simons and Keil (1995), they claimed that this understanding can exist in the absence of detailed scientific understanding of the essence. Gelman and Wellman tested children's understanding of this "inside-outside" distinction using a categorization task: Children were presented with triads of objects from which they could either choose the pair sharing the same outside (e.g., an orange and an orange balloon) or the pair sharing the same inside (e.g., an orange and a lemon). Counter to the idea that object concreteness exerts the primary influence on children's object categorization, they found that children as young as **3** years of age could correctly report both that oranges and lemons "look alike" and that oranges and orange balloons "share the same insides." Thus, young children's understanding of objects is not inevitably bound to external appearances. Rather, Gelman and Wellman argued that children's understanding of the inside-outside distinction demonstrates that nonobvious and abstract object properties also are available to children. Their findings highlighted the need for questioning the unqualified characterization of young children's thinking as being concrete.

The difficulty that children have in translating concrete representations of objects into abstract knowledge is summarized by Ball's (1992) pithy observation that "understanding does not travel through the fingertips and up the arm" (p. 47). In other words, having a concrete object does not guarantee understanding of the abstract concept. In accordance with this view, Sigel (1993) noted that attaining abstract representations requires that children be active agents in creating them. Sigel proposed that children achieve abstract representations of objects gradually through distancing acts, which "separate the child cognitively from the immediate behavioral environment" (p. 142). These distancing acts allow children to understand that symbols stand for referents by requiring that children form mental representations that are abstract rather than palpable. In other words, the translation of abstract knowledge from concrete representations is not inevitable.

In support of this view, we will present research on children's use and, in some cases, failure to understand symbols. Specifically, we will explore symbolic development through a set of tasks designed to tap children's understanding of symbol-referent relations. The difficulties children have in completing these tasks shed light on the question of whether concrete objects do, in fact, facilitate children's learning of symbolic relations.

☐ Children's Use of Scale Models

Much of our work has focused on children's understanding of a specific symbol system—scale models. This kind of symbol and the task that children are asked to perform using it have several advantages in terms of working with very young children, and the results have implications that extend to children's understanding of other kinds of symbols. The results of several studies clearly indicate that the relation between the concreteness of an intended symbol and its effect on children's comprehension of the symbolic relation is far more complex, and interesting, than previously has been assumed.

The basic task that we have used is quite simple: Children are asked to use a model to find a toy that is hidden in the room that the model represents. In most versions of the task, the model looks very much like the room; the walls are the same colors, and the model and room are furnished with highly similar (although appropriately scaled) pieces of furniture in the same spatial arrangements. For example, there is a miniature couch in the model and a full-size couch in the room; both are covered with the same fabric and in corresponding locations in the two spaces.

The tasks began with an extensive orientation that was designed to help the children grasp the relation between the model and the room. First, the experimenter showed the children the two toys that would be hidden. One toy, a miniature dog, was labeled "Little Snoopy"; the second toy, a full-size stuffed dog, was labeled "Big Snoopy." The experimenter then pointed. out the correspondences between the model and the room. The experimenter said, "This is Big Snoopy's big room; Big Snoopy has lots of things in his room." The experimenter then named each of the furniture items. Next, the experimenter pointed to the model and said, "This is Little Snoopy's little room. He has all the same things in his room that Big Snoopy has." The experimenter then labeled each of the furniture items again and pointed out the correspondence between each item in the model and the corresponding item in the room. To do this, the experimenter carried each item from the model into the room. The miniature furniture item was held next to its counterpart in the room, and the experimenter said, for example, "Look—this is Big Snoopy's big couch, and this is Little Snoopy's little couch. They're just the same."

Next, the experimenter attempted to communicate that there was a relation between actions in the model and actions in the room. For example, the experimenter told the child that, "Big and Little Snoopy like to do the same things. When Big Snoopy sits on his chair, Little Snoopy likes to sit on his chair, too." The experimenter also illustrated the correspondence by placing the toys in the appropriate positions.

The final part of the orientation involved an imitation trial in which one toy was placed in a location in either the model or the room, and the child was asked to place the corresponding toy in the correct location in the other space. For example, the experimenter placed Little Snoopy on the table in the mode! and said, "Little Snoopy is sitting on his table. Can you put Big Snoopy in the same place in his room?"

The test trials followed immediately after the orientation. On each of the test trials, the experimenter first hid the toy in one of the hiding locations in the model. The experimenter called the child's attention to the act of hiding, but not to the specific hiding location, by saying, "Look, Little Snoopy is going to hide here." The child was told that an assistant was going to hide Big Snoopy in the same place in the big room.

The experimenter and child then entered the room, and the child was asked to find Big Snoopy. On each trial, the experimenter attempted to remind the child of the relation between the model and the room by saying, "Remember, Little Snoopy is hiding in the same place as Big Snoopy." If the child could not find the toy, he or she was encouraged to continue searching at other locations, and the experimenter reminded the child again that the toy was in the "same place" as the other toy. Increasingly explicit hints were provided until the toy was found, but a search was counted as correct only if the child found the toy in the first location that he or she searched.

After the child found the toy on each trial, he or she was taken back to the model and was asked to find the miniature toy. This search provided a memory check that was critical to interpreting any difficulties that children may have had in finding the toy in the room. If the children were able to locate the miniature toy in the model, then difficulties that they encountered finding the toy in the room could not be attributed to simply forgetting where the toy was in the model. Instead, poor performance would reflect a failure to appreciate that the location of the miniature toy in the model (the symbol) could be used to find the larger toy in the room (the referent).

Several aspects of this task are important regarding the role of concreteness in children's insight into symbol-referent relations.. First, and most importantly, the model is a concrete object (or set of objects). The symbols involvedin the task are concrete. The model itself, and the furniture in the model, are tangible, manipulable replicas of larger real objects. Thus, each item is both a real object and a symbolic representation of something other than itself.

Second, successful performance requires that the child comprehend and use a symbolic relation—the relation between the model dnd the room. To solve the task, the child must understand that the location of the toy in the model specifies the location in the room. The concreteness of the model is usefulto children only if it helps them understand the stands-for relation between the model and room.

Third, children are required to solve a seemingly familiar task (searching for a toy) in a novel way. Typically, when young children search for hidden objects, they do so or, the basis of previous direct experience: like adults, they often search where they have last seen an object (DeLoache & Brown, 1983).But, to solve our task, children have to put aside this well-honed strategy and rely instead on information provided by a symbol. The model task thus allows us to examine children's first use of a novel symbol system.

Results of Research on Children's Understanding of Models

Research on children's understanding of models has revealed that, despite the apparent simplicity of the model, very young children have surprising difficulty using it. These results are summarized in Figure 9.1. Children younger than 3 years of age usually perform very poorly (only about 20% correct retrievals). The difficulty that children encounter cannot be attributed to forgetting the location of the toy that they observed being hidden. Almost all children succeed on the memory-based search in which they return to the model to retrieve the miniature toy. Thus, 2&1/2-year-olds can remember the location of the toy in the model, but they tend not to use this knowledge to find the toy in the room. Figure 9.1 also reveals that most 3-year-old children typically succeed in the standard model task (averaging over 80% correct searches).

The success of the 3-year-olds whose performance is shown in Figure 9.1 is not, however, the end of the developmental story. The difficulty that 3-year-olds have experienced in some tasks provides much of the basis for our claim that the concreteness of the model is a dual-edged sword in terms of children's performance. What, at first, seemed to be trivial manipulations often have had disastrous consequences for children's performance. The causes of the dramatic drops in children's performance illustrate the importance of understanding that the model represents the room. Concreteness can help young children to see that the model and room are similar, but there is more to appreciating the symbolic nature of the model than perceiving this similarity. We believe that the concrete nature of the model plays a rolle both in children's success and in the fragility of their performance. In this section, we discuss the results of a

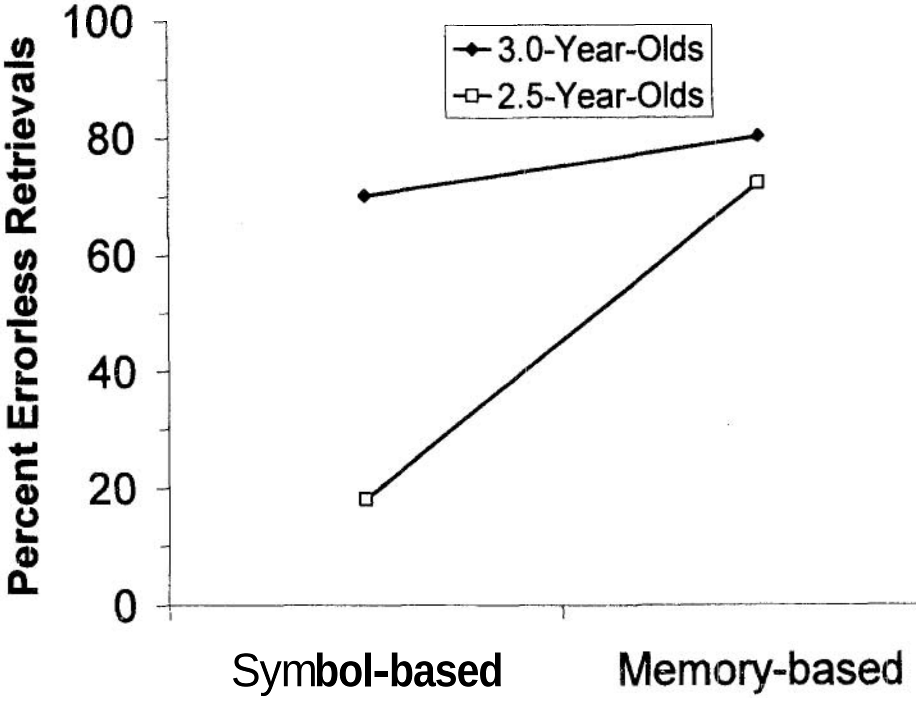

FIGURE 9.1. Children's performance in the original model study. Adapted from "Rapid Change in the Symbolic Functioning of Very Young Children," by J. S. DeLoache, 1987, Science, 238. For the symbol-based retrieval, children saw the miniature toy hidden in the rnodel and then searched for the corresponding larger toy in the roorn. For the memory-based retrieval, children returned to the model and searched for the miniature toy. Note that only the symbol-based retrieval requires that children use the relation between the model and the room to find the toy.

series of experiments that illustrate both why concreteness can help young children in some ways and why it also can hurt them.

Several different lines of research have illustrated the fragility of children's performance. For example, DeLoache, Kolstad, and Anderson (1991) demonstrated that young children':; performance depends very much on the physical similarity between the model and the room. When the furniture in the model and room were not identical, 3-year-olds performed at chance levels. Similarly, if the furniture iin the model and in the room did not occupy the same relative spatial positions, performance deteriorated substantially (DeLoache, 1995).

Subsequent studies have highlighted that the concreteness of the model lboth helps and hurts young children. On the one hand, as the results of the studies of the effects of perceptual similarity suggest, concreteness does seem to help young children to see that there is a relation between the model artd what it is intended to represent. On the other hand, subsequent research has shown that there is more to using the model than simply seeing that the model and the room look alike. The children must understand that the model is a representation of the room, and the concreteness of the model may actually make this understanding more difficuit to gain than if the model were less concrete.

Children's performance also depends very much on instructions. Recall that, in the standard model task, the experimenter explicitly points out correspondences between the model and the room. If less elaborate instructions are given, 3-year-old and even 3%-year-old children's performance is again near chance (DeLoache, 1989; DeLoache, DeMendoza, & Anderson, in press). For example, telling the children that Little and Big Snoopy's rooms are alike and that the toys are hidden in the corresponding places in the two rooms is not adequate; the experimenter also must explicitly describe and demonstrate the correspondence between the objects within them for the children to be successful. Thus, the concreteness of the model per se is no guarantee that it will be used as a symbol; young children need to be told and shown how the two are related.

With age, children become increasingly less dependent on information from the experimenter about the nature of the task (DeLoache et al., in press). Four-year-olds can succeed in the task with the reduced instructions described above, although they still need explicit information about the general model-room relation. Older children are more able to detect the relation on their own. A group of 5- to 7-year-oid children was shown the model and room and the two toys. They then observed a hiding event in the model and were asked to find the larger toy in the room (with no explanation of the relations between the spaces or the hiding events). Most of these older children inferred the "rules of the game" frorn this very minimal information and successfully retrieved the toy.

Additional research has shown that, even when children do initially grasp the relation between the model and room, they can easily lose sight of the relevance of this relation for finding the toy. In a series of studies, Uttal, Schreiber, and DeLoache (1995)showed that having to wait before using the information in the model to find the toy in the room caused 3-year-olds' performance to deteriorate dramatically. As in the standard task, the children witnessed the hiding of the toy in the modlel and then were asked to find the toy in the room. The only difference was that we inserted delays between when children saw the toy being hidden in the model and when they searched in the room. The delays were of three different lengths: 20 seconds, 2 minutes, and 5 minutes. Across the six search trials, all children experienced each of the delays twice. Differentgroups of children received the delays in one of three different orders. The groups were labeled in terms of the delay that they experienced first: the short-delay-first group had a 20-second delay first, the medium-delay-first group had a 2-minute delay first, and the long-delay-first group had a 5-minute delay first. After the initial trial, the children in each group received trials at the other delays, with delay length counterbalanced over trials.

As shown in Figure 9.2, there was a dramatic effect of the initial delay the children experienced. The long-delay-first group performed poorly, not just on the initial 5-minute delay trial, but on all subsequent trials. In contrast, the performance of the short-delay group was quite good on most of the trials. It is important to stress that the difficulty that the long-delay-first group encountered was not due simply to forgetting the location of the toy in the model during the initial delay. If this were the case, then we would expect the children to perform well on the shorter delay trials that followed the initial long delay. But,

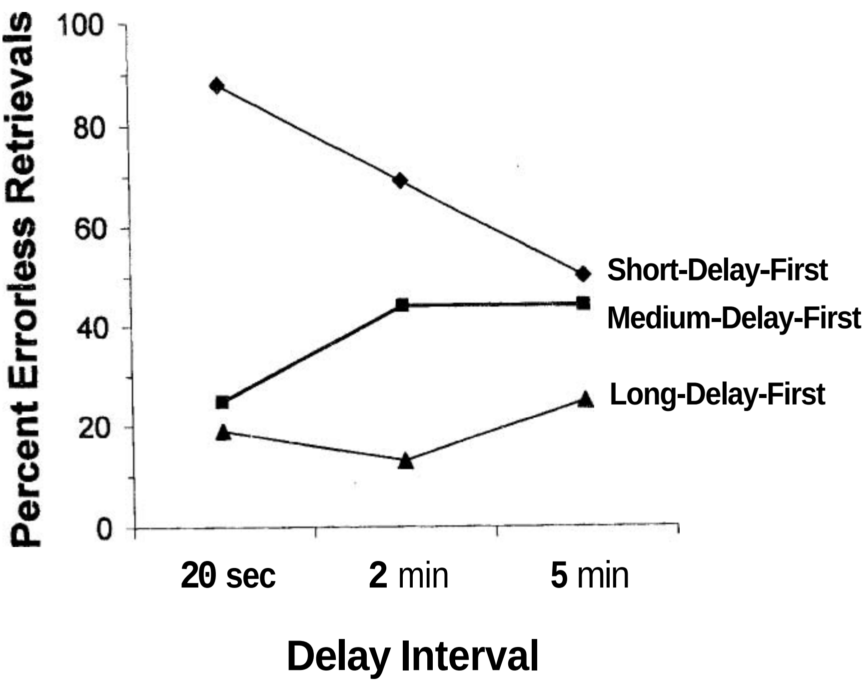

FIGURE 9.2. The effect of delay on children's use of a model. The initial long delay led to much worse performance, even on the subsequent shorter delays.

most of the children in the long-delay group performed poorly on all subsequent trials, even those trials with the short (20-s) delay that normally would give them little, if any, problem. In addition, performance on the memory-based retrievals, in which children returned to the model to find the miniature toy, were quite good, even on the long-delay trials.

Uttal et al. (1995) concluded that, during the initial long delay, the children in the long-delay-first group lost sight of the relation between the model and the room. Consequently, when they entered the room to search for the toy, they were not able to use the location of the toy in the model as a guide for searching in the room. Instead, the children conceived of the search for the toy in the model as unrelated to what they had seen in the model. In other words, the initial long delay caused the children to forget about the relevance of the model for finding the toy. Once the knowledge that the model could help was lost, the children continued to perform poorly, even on the subsequent, shorter delay trials. The chilclren apparently construed the task as two separate and unrelated searches: One search was for the larger toy in the room, and the other was for the smaller toy in the model. The 5-minute delay caused the chilclren to lose their insight into the relation between these two searches.

Support for our interpretation of the effects of the initial long delay on children's performance comes from a series of follow-up studies in which we were able to ameliorate partially the devastating effects of the initial 5-minute delay on 3-year-olds' performance. In these studies, all children experienced an initial 5-minute delay. However, after the delay, we inserted a reminder of the relevance of the model. For example, in one follow-up study, children were asked after the 5-minute delay to return to the model and find the miniature toy before they entered the room to find the larger toy. This simple manipulation led to a dramatic improvement in performance; the children now performed much better on all of the trials, including the initial 5-minute delay trial. A subsequent follow-up study indicated that simply pointing to the model and verbally reminding children that the model could help them find the toy a!so helped to inoculate children against the deleterious effects of the 5-minute delay.

The results of the follow-up studies provide additional support for the claim that, during the initial long delay, the children lost sight of the relation between the model and the room. Consequently, they had no basis of using the location of the toy in the model as a guide for search in the room. In contrast, in the follow-up studies, the reminders helped to reactivate the children's understanding of the relevance of the model and, consequently, many of the children were much less affected by the initial long delay.

In sum, the results of our research on children's use of models lead to one central conclusion: Children's success depends on their appreciation that the model is a symbolic representation of the space that it represents. When this understanding is taken away (e.g., by inserting a 5-min delay), the children have no basis for using the model. Even though the model is a highly concrete representation of the room, the concreteness alone is not enough to ensure success. Concreteness can help children to appreciate the similarity between the intended symbol and the intended referent, but perceiving this similarity alone is not the end of the story. Children must still detect the "stands-for" relation between one object and another. Concreteness alone will not ensure that children comprehend this relation.

Explaining Children's Performance: The Dual Representation Hypothesis

Why is young children's comprehension of the relation between the model and the room so fragile? Taken together, the results of our studies suggest that, ironically, the concreteness of the model is part of the problem. Our results indicate that concrete objects that are interesting and attractive as objects may actually be more difficult for young children to use as symbolic representations than materials that are less interesting as objects in their own right. This interpretation highlights the dual nature of the influence of concreteness on children's performance. While concreteness may help children to perceive similarities between symbols and their referents, it also may make it more difficult for them to think about a symbolic relation between the two.

A scale model such as the one used in our task has a dual nature; it is both a symbol and an object (or set of objects) with a very high degree of physical salience. The very features that make it highly interesting and attractive to young children as a concrete object to play with obscure its role as a symbol of something other than itself. To use a mode! as a symbol, children must achieve *dual representation* (DeLoache, 1989, 1995; Uttal et al., 1997): They must represent the model itself as an object and at the same time, as a symbol for what it represents. In the model task, the child must form a meaningful mental representation of the model as a miniature room in which toys can be hidden and found, and the child has to interact physically with it. At the same time, the child must represent the model as a term in an abstract, "stands for" relation, and he or she must use that relation as a basis for drawing inferences.

According to the dual representation hypothesis (DeLoache, 1989, 1995), the more salient a symbol is as a concrete object, the more difficult it is to appreciate its role as a symbol for something other than itself. Thus, the more young children are attracted to a model as an interesting object, the more difficult it will be for them to detect its relation to the room it stands for.

The dual representation hypothesis leads to several interesting predictions. For example, it suggests that factors that decrease children's attention to the model as an interesting object should increase their use of the model as a symbol. In one study, 2&1/2-year-old children's access to the model was decreased by placing it behind a window (DeLoache, 1998). The children could still see the location of the toy in the model, but they could have no direct contact with the model. This manipulation led to better performance. Conversely, our interpretation suggests that factors that increase children's attention to the model as an object should lead to a decrease in their use of the model. This prediction also was confirmed. Allowing 3-year-old children to play with the model for 5 to 10 minutes before they were asked *to* use it as a symbol led to a decrease in performance when children were asked to use the model to find the toy in the room (DeLoache, 1998).

Another finding that supports the dual representation hypothesis concerns children's use of photographs, rather than the model, to find the toy. Two-and-one-half-year-olds, who typically perform very poorly in the standard model task, perform much better when a photograph is substituted for the model (DeLoache, 1991; DeLoache & Burns, 1994). In some ways, a photograph could be considered less concrete than a model. Most obviously, the model is a three-dimensional representation, whereas the photograph is only two-dimensional. Nevertheless, the 2&1/2-year-olds performed much better with a photograph than their age-mates did with the model. In sum, the results indicate that a more concrete object, a model, may be more difficult to use than a less concrete object, a photograph.

Recently, DeLoache, Miller, and Rosengren (1997) have provided especially strong support for the dual representation hypothesis. In this research, 2&1/2-year-old children were led to believe that a shrinking machine could shrink (and, subsequently, enlarge) a room. The idea was that if children believe that a scale model actually is a room that has been shrunk by a machine, then there is no symbolic relation between the two spaces; to the child, the model simply is the room. Hence, dual representation is not required, so they should have no trouble reasoning between the two spaces.

Each child was first given a demonstration in which a "shrinking machine" (an oscilloscope accompanied by random computer-generated sounds described as the "sounds the machine makes while it's working") apparently caused a troll doll to turn into a miniature version of itself. The machine then supposedly "enlarged" the troll back to its original size. Next, the machine seemed to cause the "troll's room" (a tent-like room used in many previous model studies) to turn into a scale model identical to it except for size. It then enlarged the room.

The child then watched as the experimenter hid the larger troll somewhere in the portable room. After waiting while the machine "shrunk" the room, the child was asked to find the hidden toy. (The miniature troll was, of course, hidden in the same place in the model as the larger troll was in the room.) Thus, just as in the standard model task, the child had to use his or her knowledge of where the toy was hidden in one space to figure out where to search in the other. Unlike the standard task, there was no stands-for relation between the two spaces. As predicted on the basis of the dual representation hypothesis, performance was significantly better in this nonsymbolic task than in the standard model task.

Applying the Dual Representation Hypothesis to Educational Symbols

Our perspective on the dual nature of symbolic representations can be applied directly to other kinds of symbols, including those commonly used in American classrooms. The idea that concrete representations best foster young children's learning is very popular in early childhood education.

One domain to which our perspective is directly applicable is early mathematics education. Full competence in mathematics requires mastering complex concepts such as addition and subtraction and, at the same time, mastering the symbols that are used to represent these concepts. Teachers and researchers have noted, for many years, that neither is an easy task for young children. On the one hand, mathematical concepts may be difficult for young children to comprehend if they cannot understand the symbols that are used to represent these concepts. On the other hand, symbols are difficult to teach to (children who lack an understanding of the concepts the symbols represent. Not surprisingly, considerable effort has been devoted to making their initial forays into mathematics more interesting and more meaningful for young children (Hiebert & Carpenter, 1992; Stevenson & Stigler, 1992).

Many of the attempts at improvement have involved what are commonly called manipuatives. These are concrete, tangible objects such as rods and blocks that are used to represent mathematical concepts or symbols. Children are encouraged to work out mathematics problems with the tangible manipulatives, by combining units or groups of manipulatives to represent parts of the problems. The theoretical justification for the use of manipulatives is similar to, and is derived from, the more general belief in the inherent importance of concreteness in early cognition. The hope has been that, by using a concrete object, children can draw on implicit or informal mathematical concepts that might otherwise remain inaccessible. For example, by dividing a pie or candy for friends, children might gain insight, at least implicitly, to the concepts of divisions and particular fractions. The teacher could then have children recreate this process in the classroom through the use of manipulatives. In essence, the argument has been that manipulatives give teachers and children a way around the seeming opaqueness of mathematical symbols.

Manipulatives have been touted as solutions for children of a wide range of ages and ability levels; they have been offered as appropriate for all ability levels, ranging from the disabled to the gifted (Kennedy & Tipps, 1994; Tooke, Hyatt, Leigh, Synder, & Borda, 1992). Manipulatives have been incorporated into the curriculum of the National Council of Teachers of Mathematics, whose official position is as follows:

> Children come tc understand numbers meanings gradually. To encourage these understandings, teachers can offer classroom experiences in which students first manipulate physical objects and then use their own language to explain their thinking. This active involvement in, and expressions of', physical manipulations encourages children to reflect on their actions and to construct their own number meanings. In all situations, work with number symbols should be meaningfully linked to concrete materials. (1989, p. 38)

Unfortunately, research on the effectiveness of manipulatives has not confirmed the anticipated benefits. Several studies have shown, at best, inconsistent or weak advantages for manipulatives in comparison to more traditional techniques for teaching children mathematics (Hall, 1992; Resnick & Omanson, 1987; Wearne & Hiebert, 1988). Longitudinal and intensive studies of the use of manipulatives in classrooms have shown that children often fail to establish connections between manipulatives and the information that the manipulatives are intended to communicate (Sowell, 1989).

We believe that part of the reason that manipulatives have been less successful than hoped concerns problems very similar to those that younger children encounter when using a scale model. There are at least two general similarities between what is required to succeed in our model task and to what is required to effectively use a manipulative. The first is that the relation between a manipulative and what it is intended to represent may not be transparent to young children. In other words, the concreteness of a manipulative (or of our model) does not guarantee that children will understand that it is intended to represent something other than itself. Our model is extremeiy concrete, and parents are amazed when it proves difficuit for intelligent, interested children. We believe that similar issues may arise when older children are asked to use manipulatives; to the teacher, the relation between the manipulative and a more abstract concept may be simple and direct but this relation may be, and may reinain, obscure to young children, particularly if the relation is not pointed out explicitly.

The second similarity between children's difficulties with our model and with manipulatives is that dual representation is relevant to both. As was true of our model, manipulatives have a dual nature; they are intended to be used as representations of something else, but they also are objects in their own right. In the next section, we review some difficulties that children encounter when using manipuiatives, difficulties that parallel younger children's problems with our scale model and that are consistent with the dual representation perspective.

Children Often Fail to Grasp the Relation Between Manipulatives and What the Manipulatives Are Intended to Represent

From a teacher's point of view, the goal of using a manipulative is to provide support for learning more general mathematics concepts. However, this is no guarantee that children will see the manipulative in this way. Previous work on the use of manipulatives has documented numerous examples of mismatches between teachers' expectations and students' understandings. Even when young children do learn to perform mathematical operations using manipulatives, their knowledge of the two ways of solving the problems may remain encapsulated; that is, children often fail to see the relation between solving mathematics problems via manipulatives and solving the same or similar problems via abstract symbols (see Uttal et al., 1997, for a review).

Evidence that children often fail to draw connections between manipulatives and more traditional forms of mathematical symbols comes from Resnick and Omanson's (1987) intensive studies of children's use of manipulatives and their understanding of mathematical concepts. Resnick and Omanson systematically evaluated third-grade children's ability to solve problems both with and without rnanipulatives. Much of the work involved Dienes blocks, which are a systematic set of manipulatives that are designed to help children acquire understanding of base 10 concepts. Most of the children understood and appeared to enjoy working with the blocks.

Unfortunately, however, the children's ease with and knowledge of the blocks was not related to their understanding of similar kinds of problems expressed in more formal mathematical terms. The children did not relate approaches they had used to solve problems with manipulatives to the solution of similar problems involving written symbols. For example, children who were successful in using Dienes blocks to solve subtraction problems involving two or three digits had trouble :solving simpler written problems. Indeed, the child who performed best with the Dienes blocks performed worse on the standard problems. Clearly, success with a manipulative did not guarantee success with written symbols; in fact, success with one form of mathematics expression was unrelated to success with the other.

Other researchers have provided additional evidence of the nonequivalence of concrete and more abstract forms of mathematical expressions. For example, Hughes (1986) investigated young elementary school children's ability to use simple blocks or bricks to solve addition and subtractiori problems What is most interesting about this study for the current discussion is that the children were explicitly asked to draw connections between solutions involving concrete objects and those involving more abstract, written problems. The children were asked to use the bricks to represent the underlying concepts that were expressed in the written problems. For example, the children were asked to use bricks to solve written problems, such as 1 + 7 = ?.

Overall, the children performed poorly. Regardless of whether they could solve tho written problems, they had difficulty representing the problems with the bricks. Moreover, the children's errors demonstrated that they failed to appreciate that the bricks and written symbols were two alternate forms of mathematical expression. Many children took the instructions literally, using the bricks to physically "spell out" the written problems (Figure 9.3). For example, they might make a line of bricks to represent the "1" and two intersecting lines to represent the "+" and so on. These results again highlight that children may treat solutions involving manipulatives and those involving written mathematical symbols as cognitively distinct entities.

The research on children's understanding of manipulatives also highlights the conditions under which manipulatives are likely to be effective. Specifically, the results of several studies suggest that rnanipulatives are most effective when they are used to augment, rather than to substitute for, instructions involving written symbols. In these cases, teachers have drawn specific connections between children's use of a manipulative and the related expression of the underlying concept in written form. For example, consider Wearne and Hiebert's (1988) program. It focuses on fractions, but the results are relevant to other mathematical concepts. At all stages of the program, the teacher draws specific links between rnanipulatives and written symbolic expressions. The manipulative is used as a bridge to the written expressions rather than as a substitute or precursor for written symbols. As a result, the manipulative scaffold:; the learning of the written symbols by gradually leading children away from the concrete properties of the manipulative to the more foreign properties of the written symbols. Thus, the focus of this and similar successful programs is on the relation between manipulatives and other forms of mathematics expression.

Letters as a Symbol System. The questions raised in this chapter regarding children's acquisition of symbols also are relevant to the early development of reading. In learning to read, children must master the relation between an abstract syinbol system and its referents. The assumption of a concrete-to-abstract shift leads to the further assumption that young children would benefit if the abstract system of letters were transformed into a concrete form they could more readily understand.

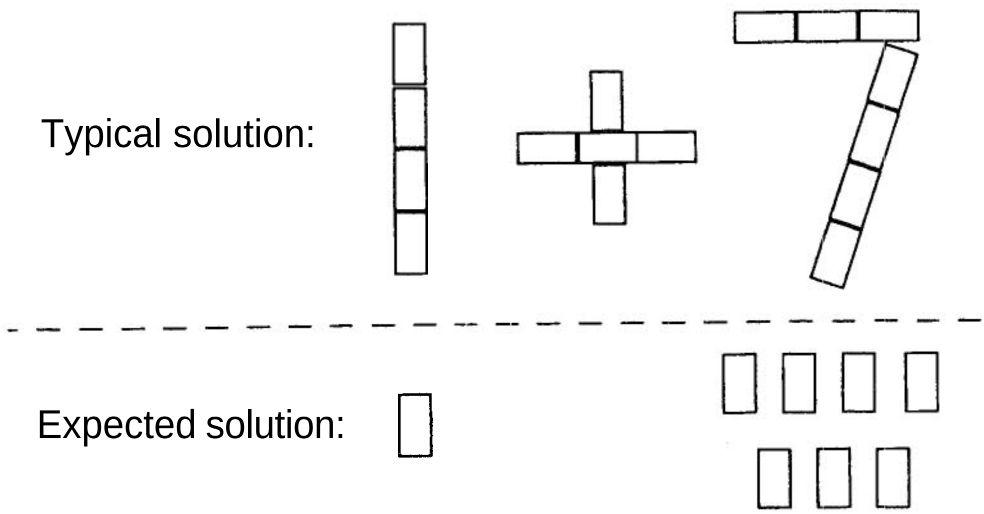

FIGURE **9.3.** An example of how children might use small bricks to represent the problem 1 + 7 = 8. From Children and Numbers: Difficulties *in* Learning Mathematics, by M. Hughes, 1986, pp. 99–103. Copyright 1986 by Basil Blackwell. Adapted with permission. The children often copied written problems with the bricks rather than using the bricks as an alternate representational system.

Given that children's use of symbols does not unequivocally benefit from concreteness, however, challenges to the concrete-to-abstract shift concept may change ideas of how the development of reading is best supported. Research on notational symbol systems provides further evidence againist the idea that concrete and abstract thinking develop independently. Understanding letters is difficult because letters are noniconic symbols. Unlike pictographs, there is nothing inherent in the structure of letters that reflects what they represent. In essence, understanding letters as notational symbols requires that children appreciate nonanalogous, noniconic symbolic relations (Bialystok, 1992).

Bialystok (1992) proposed that children must relinquish their hold on the specific perceptual properties of objects to understand them as symbols. Symbol acquisition emerges in three stages as children's initially fragile understanding of symbols becomes more flexible. Children may first learn a set of symbols without understanding their relation to what they represent. For example, they may be capable of verbally reproducing a sequence of symbols (e.g., counting in a series or reciting the alphabet). 'They may then begin to observe the relation of these objects to their referents. In this second stage, children tend to assume that the relations between symbols and referents are iconic and analogous. For example, they may believe that the word "ant" is shorter than the word "elephant" because ants are smaller than elephants. Similarly, Spanish and Italian children associate bigger words with bigger objects, in spite of the fact that this relationship is even less perfect in both of these languages (Ferreiro, 1988, as cited in Bialystok, 1992). Both Spanish and Italian languages use the suffix "ita" to demarcate diminutives of root words, so that longer words actually denote smaller objects. When children finally acquire full symbolic competence in Bialystok's third stage, they are capable of understanding that symbols may be noniconic and nonanalogous (e.g., "car" is shorter than "banana," even though cars are larger than bananas). Thus, Elialystok (1992) demonstrated that the acquisition of symbols such as letters and numbers occurs in a gradual three-step process, not as an abrupt concrete-to-abstract shift.

Once children know the correspondences between the written forms (graphemes) and auditory forms (phonemes), they have the requisite knowledge to read and write any word in the language (Adams, 1990). Learning individual grapheme-phoneme correspondences, however, is neither easy nor a guarantee that children will learn to read. In fact, Landsmann and Karmiloff-Smith (1992) found that children's understanding of letters as part of a notational symbol system does not necessarily co-occur with their understanding of how the letters are used in referential communication. They asked children age 4 through 6 to invent nonletters, nonnumbers, and nonwords. Children in all age groups imposed different constraints on what qualified as nonletters and nonnumbers, demonstrating their understanding that letlers and numbers were separate domains of symbols but also that they are not in the same domain as drawings. For example, one child produced "tttt" when asked to generate a nonword (p. 297). Only the older children, however, understood that symbols serve a referential role as well as a notational role. Rather than simply using strings of repeated letters to create nonwords, 5- and 6-year-olds generated nonwords that were unpronounceable and, thus, could serve no referential function.

If, in fact, making letter learning concrete were the best way to foster reading skills, one might expect that children would quickly learn to identify symbols that are embedded in real-life situations and require communication. Landsmann and Karmiloff-Smith (1992) suggested that learning the symbolic role of letters is a task in its own right and separate from children's use of these symbols in reading or in communication. This interpretation suggests that, although the role of symbols in referential communication is more salient and more concrete to young children, it does not play a prominent role in children's early understanding of symbols as a notational system.

Given the importance of the alphabet and the problems children may have in learning it, parents often turn to other means of making letter learning more concrete. Concrete manipulatives such as alphabet blocks potentially can provide a tactile means of teaching reading in much the way that Dienes blocks allow hands-on learning of mathematics. Like mathematics manipulatives, alphabet blocks transform the abstractness of graphemes and phonemes into familiar, perceptually rich objects. Although the use of manipulatives for reading instruction has not been investigated as the use of manipulatives in math education has been, it seems likely that similar caution is appropriate. Simply putting the letters of the alphabet on colorful blocks does not guarantee that children will learn to use them for spelling rather than as building blocks.

According to the dual representation hypothesis, attempts to make alphabet blocks colorful and engaging as objects might detract from the child seeing the letters on them as symbols. The physical features or concreteness of the blocks actually may obfuscate the symbol-referent relation. Alphabet blocks, for example, typically are constructed in different colors, which facilitates children's perceptual differentiation of different letters when they are learning the alphabet early on. The elaboration of individual letters is similarly evident in the topical organization of Sesame Street, which typically focuses on only two letters of the alphabet per episode (i.e., "This episode brought to you by the letter 'E'") and in different skits used to interest children in learning their letters (e.g., the letter beauty pageant). Such attempts to make individual letters interesting may distract from the collective function the letters :serve within the notational svstem as a whole. Emphasizing letters as perceptually salient objects in their own right may, in fact, make it more difficult to see each letter as being a component of a word and as serving an equivalent notational role in the alphabet.

One might predict, however, that alphabetic manipulatives may not share all of the problems associated with mathematics manipulatives. Aithough individual letters, like numbers, have different critical features such as curves and straight lines, letter blocks are the same size and, thus, equate the physical characteristics of different letters to some extent. It might be argued that one of the steps to understanding the notational function of letter symbols is understanding that individual letters are functionally equivalent when they constitute words. In alphabet blocks, children can create strings of letters simply by combining various blocks, without being distracted by differences in letter size. Interestingly, the rnistake that children make by using Dienes blocks to spell out mathematics equations is precisely the correct way to use alphabet blocks. In this regard, alphabet blocks may be more iconic symbols for letters than manipulatives are for mathematical equations, because collections of blocks are assembled in a way that is physically anaiogous to how words are put together.

☐ Implications

Our review of children's understanding and use of concrete symbols has several important implications for the use of concrete objects in educational contexts. These implications ernphasize that concrete objects can help young children understand symbol-referent relations, but that the ultimate goal must remain to help children comprehend more abstract relations.

1. Concreteness is not a panacea. The most important implication of our review is that concreteness alone is not sufficient to help children learn symbolic relations. Under the right circumstances, concrete

objects can help children make initial insights into symbol-referent relations. I-Iowever, if the objects are not explicitly linked to less concrete representations, the knowledge about their use is likely to remain encapsulated. The best concrete object symbols may well be ones that possess elements of their own destruction, designed to become less necessary or relevant as children come to understand the more abstract representation.

2. Concreteness is a two-edged sword. Our review of research suggests that concreteness may both help and hinder young children's appreciation of the relations between symbols and their intended referents. On the one hand, concreteness, in some ways, can facilitate children's use of symbols. For one thing, concrete objects are generally attention getting and more interesting to young children. For another, some symbol-referent relations are more obvious with concrete objects as symbols. For example, children often can solve mathematics problems with manipulatives before they can solve similar problems with more abstract mathematics symbols.

On the other hand, the characteristics that make concrete objects interesting to young children may simultaneously make them difficult for young children to move beyond them. If an object is too interesting as a thing in it: elf, then children may have more difficulty using it as a symbol for something else. This suggests that the best concrete objects may be those that are interesting enough as objects to engage young children's attention, but not so attractive as to make it difficult to focus on the relation between the object and its intended referent. Thus, manipulatives that are selected specifically because they are interesting and attractive to young children actually may be counterproductive.

Observations of manipulative use in other countries have supported the idea that a good mainpulative is not necessarily an inherently interesting object. For example, in Japan, children use the same set of manipulative throughout the early elementary school years. Stevenson and Stigler (1992), who have conducted several cross-national comparisons of mathematics achievement in Asia and the United States, have observed the following:

> Japanese teachers. . .use the items in the math set repeatedly throughout the elementary school years. . . . American teachers seek variety. They may use Popsicle sticks in one lesson, and marbles, Cheerios, M&M's, checkers, poker chips, or plastic animals in another. The American view is that objects should be varied in order to maintain children's interest. The Asian view is that using a variety of representational material:; may confuse children, and thereby make it more difficult for them tc use the objects of the representation and solution of mathematics problems. Multiplication is easier to understand when the same tiles are used as were used when the children learned to add. (pp. 186–187)

3. Concrete objects are not a substitute for instruction. Our review clearly indicates that instructions are a critical part of learning about symbol-referent relations, regardless of whether the symbols are concrete or abstract. As the research on children's use of mathematics manipulatives has clemonstrated, concrete objects assist children's understanding of more abstract symbols only when explicit links are made between the two forms of symbolic expressions. Children cannot be expected to make the connections on their own. Concrete objects should be thought of as a potentially helpful aid to instruction, but only in combination with instruction in their use.

Conclusions

Mastering symbol systems is one of the most important and challenging; tasks of childhood. It is not surprising that teachers and parents have attempted to make the task easier by providing children with sets of concrete objects that seem, at least ostensibly, to make the task of learning symbol systems easier. Despite the criticisms that we have raised in this chapter, we do not believe that the original assumptions regarding the value of concreteness for learning symbol-referent relations are inherently wrong. Concrete objects can help young children, at least initially, to gain insight into the basic relation between a symbol and its referent. Concrete objects can provide a scaffold on which an understanding of more abstract relations can be built.

The problems with concrete objects that we have discussed arise only when the concrete object is used as a substitute, rather than a scaffold, for an understanding of more abstract symbols. Objects that are particularly attractive as objects may be particularly harmful in this regard, by focusing children's attention on the objects themselves rather than on what the symbols were intended to represent. The impulse to make objects interesting and engaging to young children actually may detract from the ultimate goal of helping children understand how abstract symbols relate to concepts. In sum, it is important to consider both the advantages and disadvantages of using concrete objects to help children learn symbolic relations. Concrete objects are useful to the extent that they assist in, rather than substitute for, the learning of abstract concepts.

References

Adams, M. J. (1990). Beginning to read: Thinking and learning about print. Cambridge, MA: MIT Press.

Ball, D. L. (1992). Magical hopes: Manipulatives and the reform of math education. American Educator, 26, 14–18.

Bialystok, E. (1992). Symbolic representation of letters and numbers. Cognitive Development, 7, 301–316.

Bruner, J. S. (1966). Toward a theory of instruction. Cambridge, MA: Belknap Press.

Bruner, J. S., Goodnow, J. J., & Austin, G. A. (1956). A study of thinking. New York: Wiley.

DeLoache, J. S. (1987). Rapid change in the symbolic functioning of very young children. Science, 238, 1556–1557.

DeLoache, J. S. (1989). Young children's understanding of the correspondence between a scale model and a larger space. Cognitive Development, 4, 121–139.

DeLoache, J. S. ('1991). Symbolic functioning in very young children: Understanding of pictures and models Child Development, 62, 736-752.

DeLoache, J. S. (1995). Early symbol understanding and use. In D. Medin (Ed.), The psychology of learning and motivation (Vol. 33, pp. 65–114). New York: Academic Press.

DeLoache, J. S. (in press). Dual representation and young children's use of scale models. Child Development.

DeLoache, J. S., & Brown, A. L. (1983). Very young children's memorv for the location of objects in a large scale environment Child Deelopment, 24, 888-897.

DeLoache, J. S., & Burns, N. M. (1994). Early understanding of the representational function of pictures Cognition 52, 83-110.

DeLoache, I. S., DeMendoza, O A. P., & Anderson, K. M. (in press). Multiple factors in early symbol use: The effect of instructions, similarity, and age in understanding a symbol-referent relation. Cognitive Deverlopment.

DeLoache, J. S., Kolstad, V., & Anderson, K. (1991). Physical similarity and young children's understanding of scale models. *Child Development, 62,* 111–126.

DeLoache, J. S., Miller, K. F., & Kosengrcn, K. S. (1997). The credible shrinking room: Very young children's performance with symbolic and nonsymbolic relations. *Psychological Science,* 8, 308–313.

Dias, M. G., & Harris, P. L. (1988). The effect of make-believe play on deductive reasoning. *British Journal of Developmental Psychology,* 6, 207–221.

Dins, M. G., & Harris, I. L. (1990). The influence of the imagination on reasoning by young children. *British Journal of Developmental* Psychology, 8, 305–318.

Gelman, S. A., & Wellman, H. M. (1991). Insides and essences: Early understandings of the non-obvious. *Cognition,* 38, 213–244.

!Iiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research* on *mathematics teaching* nnd learning, (pp. 65–97). New York: Macmillan.

Hughes, M. (1986) *Children* and number: *Difficulties* in *learning mathematics.* Oxford, England: Basil Blackwell.

Inhelder, B., & Piaget, J. (1958). *The growth of logical* thinking *from childhood* to *adolescence: An essay on the construction of formal* operational *structures.* New York: Basic Books. (Original work published 1955)

Kennedy, L. M., & Tipps, S. (1994). Guiding *children's learning* of mathematics (7th ed.). Belmont, CA: Wadsworth.

Landsmann, L. T., & Karmiloff-Smith, A. (1992). Children's understanding of notations as domains of knowledge versus referential-communicative tools. *Cognitive Development,* 7, 287–300.

Montessori, M. (1917). *The advanced* Moiztesorri method. New York: Frederick A. Stokes.

Moshman, D., & Franks, 8. A. (1986). Development of the concept of inferential validity. *Child Development,* 57, 153–165.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for* school *mathematics.* Reston, VA: Author.

Piaget, J. (1951). Play, *dreams,* and *imitation* in childhood. New York: Norton.

Resnick, L. B., & Omanson, S. F. (1987). Learning to understand arithmetic. In R Glaser (Ed.), *Advances* in *instructional psychology* (Vol. 3, pp. 41–96). Hillsdale, NJ: Erlbaum.

Sigel, I. E. (1993). The centrality of a distancing model for the development of representational competence. In R. R. Cocking & K. A. Rennigner (Eds.), *The development* and *meaning* of psychological *distance* (pp. 141–158). Hillsdale, NJ: Erlbaum.

Simons, D. J., & Keil, F. C. (1995). An abstract to concrete shift in the development of biological thought: The *insides* story. *Cognition,* 56, 129–163.

Sowell, E. J. (1989). Effects of manipulative materials in mathematics instruction. *Journal for Research in Mathematics Education, 20,* 498–505.

Stevenson, H. W., & Stigler, J. W. (1992) *The learning gap: Why our schools are failing and what we can learn from Japanese and Chinese education.* New York: Summit Books.

Tooke, D. J., Hyatt, B., Leigh, M., Synder, B., & Borda, T. (1992). Why aren't manipulatives used in every middle school mathematics classroom? *Middle School Journal, 24,* 61–62.

Uttal, D. H., Schreiber, J. C., & DeLoache, J. S. (1995). Waiting to use a symbol: The effects of delay on children's use of models. *Child Development,* 66, 1875–1891.

Uttal, D. H., Scudder, K. V., & DeLoache, J. S. (1997). Manipulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. *Journal of Applied Developmental Psychology, 18,* 37–54.

Vygotsky, L. (1962). *Thought and language* (E. Hanfmann & F. Vacar, Trans.). Cambridge, MA: MIT Press. (Original work published 1934)

Wearne, D., & Hiebert, J. (1988). A cognitive approach to meaningful mathematics instruction: Testing a local theory using decimal numbers. *Journal for Research in Mathematics Education, 29,* 371–384.

Werner, H., & Kaplan, B. (1963). *Symbol formation: An organismic approach to language and the expression of thought.* New York: Wiley.