

Concreteness and Symbolic Development

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INTRODUCTION

The ability to understand and use symbols is one of the defining characteristics of being human. Symbols allow us to think about information that is not available to direct sensory experience. Symbol systems such as language also allow us to communicate with others and thus provide the foundation for learning. Similarly, numbers allow us to think about and mentally manipulate abstract representations rather than having to rely on the actual physical quantities. It is not surprising that the development of symbolic capacity is an important hallmark in almost all theories of cognitive development.

Much research on symbolic development is motivated by the assumption that young children's thinking is inherently concrete in nature, and that their thinking focuses only on immediately perceptible concepts (Bruner, 1966; Piaget, 1951; Vygotsky & Kozulin, 1986; Werner & Kaplan, 1963). In contrast, older children are more able to think about abstract concepts that are not tied to the concrete, perceptible properties of objects that they can see or feel. Put simply, the general notion that concrete thinking precedes abstract thinking is characteristic of most theories of development.

The general assumption that young children's thinking is inherently concrete in nature has had a tremendous effect on the development of educational curricula and materials. Many researchers and educators believe that the best way to help young children learn to understand the abstract properties of symbolic relations is to first make the symbols less abstract and more concrete (Ball, 1992; Clements & McMillen, 1996; Montessori, 1917). For example, Bruner (1966) suggested that the goal of early education should be to "empty the concept of specific sensory properties [in order to] grasp its abstract properties" (p. 65). This assumption has led to the development of a wide variety of educational materials that are specifically designed to appeal to young children's preference for concrete, tangible objects. Examples include letter blocks, number magnets, and formal manipulative systems, such as Dienes Blocks and Cuisenaire Rods. Many early childhood educators assume that these sorts of materials are the best, or even only, way for young children to learn. The assumption has been that "Concrete is inherently good; abstract is inherently not appropriate— at least at the beginning, at least for young learners" (Ball, 1992, p. 16).

The primary purpose of this chapter is to reexamine the focus on concreteness in cognitive development and early education. We will question both the theoretical background of the assumption as

well as its educational implications. We will show that the characterization of development in terms of a shift from concrete to abstract is an oversimplification; there are situations in which very young children seem capable of abstract reasoning, and there are other situations in which older children's thinking is highly concrete. We also question the assumption that concrete objects should necessarily provide the foundation for young children's learning of symbolic relations. In some cases, the use of an attractive concrete object may actually have a negative effect because it may focus children's attention more on the object itself rather than on what the object is intended to represent. These claims are based on a review of both classic and current literature on the development of children's understanding of important symbols, including letters, numbers, mathematical symbols, and scale models. We begin by considering the historical and theoretical origins of the commonly accepted belief that young children's thinking is inherently concrete and that early childhood education therefore should focus on the use of concrete objects. We also review recent theoretical and empirical work that has demonstrated that these assumptions may not always be correct.

TRADITIONAL APPROACHES: THE CONCRETE-TO-ABSTRACT SHIFT

Development often has been characterized as children's struggle to transcend their shallow and short-sighted view of the world (Bruner, Goodnow, & Austin, 1956; Piaget, 1951; Werner & Kaplan, 1963). In classic developmental theories, the acquisition of symbolic competence is seen to proceed through a concrete-to-abstract shift: the progression from thinking that is rooted in concrete reality to thinking that is less constrained by context. Sigel (1993) described this developmental progression as the child's attempt to "separate him- or herself mentally from the ongoing here and now, and project him- or herself to some other temporal plane (past or future or the nonpalpable present), in turn transforming the received communication into some symbol or sign system" (p. 142). Eventually, children's mental representations are no longer directly linked, in an iconic fashion, to the information that they originally experienced. Instead, older children are able to represent information more abstractly, so that the information is now only distantly related to how it was experienced initially.

Almost all classic theories of cognitive development have appealed to the idea that young children's thinking is inherently concrete. For example, Piaget (Inhelder & Piaget, 1958; Piaget, 1951) suggested that the development of the ability to reason in terms of abstract, hypothetical propositions, without reference to more concrete information, was the end point or goal of cognitive development. Piaget found that concrete operational children had trouble reasoning about false propositions that involved relations that could not exist in the real world. For example, if concrete operational children are given the statements, "If mice are bigger than dogs and dogs are bigger than elephants," they typically cannot deduce "then mice are bigger than elephants." These sorts of problems require that children reason abstractly about the relations as given, rather than about the actual relations in the world (Werner & Kaplan, 1963). Concrete operational children fail because there is no concrete basis from which to reason about and solve the problem.

Other prominent theorists have also characterized development in terms of a shift from concrete to abstract. For example, in studies of early categorization, Bruner et al. (1956) described conceptual development as a perceptual-to-conceptual shift; children first think of objects only in terms of the properties directly available to their senses but eventually begin to consider abstract properties of objects. For example, children may think that birds and bats are in the same category because they look similar and because they both fly. With development, children become able to categorize objects and living things more on the basis of abstract and nonobservable information. Consequently, they now realize that bats and birds should be in separate categories, and that a creature that does not fly, such as a penguin, may nevertheless belong in the bird category. The developmental transition is thus from a reliance on concrete and perceptible properties to more abstract and less observable ones.

Some of Vygotsky's writings are reminiscent of the concrete-to-abstract shift. Specifically, Vygotsky (Vygotsky & Kozulin, 1986) conducted two lines of work that were motivated by this general assumption. First, he suggested that young children's classification is inherently thematic in nature. Thematic categories (e.g., rabbit and carrot) are based on highly concrete, salient properties that bind objects

or living things together in a common setting, rather than on the underlying and abstract relations; they are developmentally primitive. The more developmentally advanced form of categorization (e.g., carrot and potato) is based on *taxonomic* properties. To think about objects in this way, children must learn to look beyond the concrete and perceptually similar characteristics in favor of deeper but less obvious similarities. Second, Vygotsky also pointed out the important role of concreteness in symbolic play. Young children's pretend play often involves the substitution of concrete objects for something else in the real world (e.g., a stick for a horse). He suggested that the use of concrete objects in this way was an early form of symbolization. In the context of the game, children are less bound to the properties of the objects and feel comfortable substituting the objects for something else. Pretend play thus serves the important function of helping children to see that the physical object can be thought of in a different way, as a representation of something else.

Werner and Kaplan (1963) provided what is perhaps the most specific articulation of the relation between concreteness and symbolic development. They argued that development involved a shift from holistic to analytic thinking. Young children initially focus on "physicochemical stimuli" from the environment. By this Werner and Kaplan meant that young children interpret stimuli in terms of their concrete, physical properties. Eventually, children transform stimuli and interpret them as "stimulus-signs or signals" (p. 9). For example, a young child might interpret the letter "A" as two diagonal lines and a crossbar. An older child instead interprets the letter as being related to language, even if he or she does not precisely know how this relation works (see Bialystok et al., 2000).

ALTERNATE PERSPECTIVES: DOES DEVELOPMENT ALWAYS PROCEED FROM CONCRETE TO ABSTRACT?

The notion that young children's thinking is inherently concrete in nature is not universally accepted. For example, researchers have recently presented evidence that even infants are capable of thinking about abstract concepts. Other researchers have challenged the notion that development proceeds from concrete to abstract, suggesting that in some cases the opposite could be true. In this section we briefly summarize these findings and theoretical perspectives.

Abstract Concepts in Infants

Recent research on cognitive development in infancy provides an important challenge to the idea that development proceeds from concrete to abstract. Several lines of research have revealed that infants can interpret movements or actions in terms of abstract concepts. For example, Quinn and colleagues (Quinn, 2003; Quinn, Adams, Kennedy, Shettler, & Wasnik, 2003) have found that infants interpret the position of objects in terms of abstract spatial concepts, such as above, below, and between. By 10 months of age, infants will notice if an object is moved from between two lines to above or below one of the lines, even if the object itself changes. Their judgments of spatial position therefore are not tied to the concrete properties of the objects themselves but are instead based on more abstract concepts such as "between." Likewise, young children are capable of interpreting another person's actions in terms of the goals or intentions that motivate those actions. For example, infants will interpret the hand motion of another person as related to the goal of opening a box to obtain a toy (Gergely, Nadasdy, Csibra, & Biro, 1995; Woodward, 2003; Woodward & Sommerville, 2000). After observing a person opening a box to obtain a toy, they are more surprised when the hand moves to a different box than when the hand moves in a different pattern to the same box. Moving to a different box indicates that the person has a different goal in mind. The infants appear to understand the association between where the hand moves and what the person's intent is. This sort of abstract knowledge allows infants to interpret an action, for example, "as getting a drink of milk rather than grasping a milk carton" (Woodward & Sommerville, 2000, p. 76).

Other research has demonstrated that preschoolers use abstracts concepts as a basis for reasoning, inference generation, and problem solving. For example, Gelman and colleagues (Gelman, 2000, 2003; Gelman & Wellman, 1991) have suggested that children understand that certain objects have an

internal "essence" that is distinct from the outward appearance of the objects. Gelman has suggested that this understanding can exist in the absence of detailed scientific understanding of the essence. Gelman and Wellman (1991) tested children's understanding of this "inside-outside" distinction using a category induction task. Children 3 and 4 years of age were shown a target object and two choice objects. They were asked (1) to choose which of the two choice objects "looks most like" the target, and (2) to choose which of the two "has the same kinds of insides" as the target. For example, children were presented with triads of objects from which they could either choose the pair sharing the same outside (e.g., an orange and an orange balloon) or the pair sharing the same inside (e.g., an orange and a lemon). Counter to the idea that object concreteness exerts the primary influence on children's object categorization, they found that children as young as 3 years of age could correctly report both that oranges and orange balloons "look alike" and that oranges and lemons "share the same insides."

Thus, young children's understanding of objects is not inevitably bound to external appearances. Rather, children's understanding of the inside-outside distinction demonstrates that nonobvious and abstract object properties also are available to children (Gelman, 2003). These findings highlight the need to question the unqualified characterization of young children's thinking as being concrete. Children's performance on these sorts of tasks forms part of the basis for Gelman's (2003) claim that young children are *essentialists*. Even young children reason about animals and other entities in terms of abstract-like principles that define their essential characteristics. What matters most to young children, for example, is what is inside an animal, rather than its superficial appearance.

Simons and Keil (1995) presented the most radical reformulation to date of the developmental relation between abstract and concrete thinking. They suggest that the development of children's thinking may, in fact, proceed from abstract to concrete. They argued that very young children may first reason at an abstract level because they lack specific knowledge about objects and events. For example, a child explaining the function of a camera might initially discuss a camera's ability to capture a single point in time, such as its ability to record the moment when she blew out the candles on her birthday cake. Simons and Keil argued that this functional understanding of the camera can precede a more concrete and mechanistic understanding of how light enters the lens and how the various parts of the camera interact. In Simon and Keil's (1995) words, "Although ignorance of the physical components of a system may preclude a concrete explanation for the system's behavior, it is quite possible to generate a principled, abstract explanation without any knowledge of the physical components" (p. 131).

In summary, the notion that young children's thinking is inherently concrete in nature has been challenged in many ways. There is evidence that young children (perhaps even infants) can think in terms of abstract concepts, and there is also evidence that development may sometimes proceed in the opposite direction—from abstract to concrete. In the next section, we consider the relevance of these findings for research on symbolic development. The difficulties children have in using certain kinds of symbols shed light on the question of whether concrete objects do, in fact, facilitate children's learning of symbolic relations.

CONCRETENESS, SYMBOLIC DEVELOPMENT, AND CHILDREN'S USE OF SCALE MODELS

Much of our work on symbolic development has focused on children's understanding of a specific symbol system—scale models. Studying children's understanding of scale models has provided important windows onto the process of symbolic development and the effects of concreteness on symbolic understanding. The results of several studies clearly indicate that the relation between the concreteness of an intended symbol and its effect on children's comprehension of the symbolic relation is far more complex, and interesting, than has been assumed previously.

The Scale Model Task

Our task (DeLoache, 1987) for studying symbolic development is quite simple: We ask young children to use a scale model to find a hidden toy. Usually, the model and the room look very much alike except for size; the walls are the same colors, and the furniture in the model and the room are upholstered

with the same fabric. Moreover, there is a high degree of spatial similarity as well. All of the objects in the model are usually placed in the same relative spatial positions as in the room.

We begin by explaining the task and by orienting children to the relation between the model and the room. First, the experimenter points out the two toys that will be hidden. One toy, a miniature dog, is labeled "Little Snoopy"; the second toy, a full-size stuffed dog, is labeled "Big Snoopy." The experimenter then demonstrates the correspondences between the model and the room. The experimenter says, "This is Big Snoopy's big room: Big Snoopy has lots of things in his room? The experimenter then names each of the furniture items. Next, the experimenter points to the model and says, "This is Little Snoopy's little room. He has all the same things in his room that Big Snoopy has." The experimenter then labels each of the furniture items again and highlights the correspondence between each item in the model and the corresponding item in the room. The experimenter carries each item from the model into the room. The miniature furniture item is held next to its counterpart in the room, and the experimenter says, for example, "Look—this is Big Snoopy's big couch, and this is Little Snoopy's little couch. They're just the same."

Next, the experimenter attempts to communicate that there is a relation between actions in the model and actions in the room. For example, the experimenter tells the child that "Big and Little Snoopy like to do the same things. When Big Snoopy sits on his chair, Little Snoopy likes to sit on his chair, too." The experimenter also illustrates the correspondence by placing the toys in the appropriate positions.

The test trials follow immediately after the orientation. On each of the test trials, the experimenter first hides the toy in one of the hiding locations in the model. The experimenter calls the child's attention to the act of hiding, but not to the specific hiding location, by saying, "Look, Little Snoopy is going to hide here." The child is told that an assistant is going to hide Big Snoopy in the same place in the big room.

The experimenter and child then enter the room, and the child is asked to find Big Snoopy. On each trial, the experimenter attempts to remind the child of the relation between the model and the room by saying, "Remember, Little Snoopy is hiding in the same place as Big Snoopy." If the child cannot find the toy, he or she is encouraged to continue searching at other locations, and the experimenter reminds the child again that the toy is in the "same place" as the other toy. Increasingly explicit hints are provided until the toy is found, but a search is counted as correct only if the child finds the toy in the first location that he or she searches.

After the child finds the toy on each trial, he or she is taken back to the model and is asked to find the miniature toy. This search provides a memory check that is critical to interpreting any difficulties that children may have in finding the toy in the room. If the children are able to locate the miniature toy in the model, then difficulties that they encounter finding the toy in the room cannot be attributed to simply forgetting where the toy is in the model. Instead, poor performance reflects a failure to appreciate that the location of the miniature toy in the model (the symbol) can be used to find the larger toy in the room (the referent).

Several aspects of this task and of our results are important in regard to the role of concreteness in children's insight into symbol-referent relations. First, and most importantly, the symbols involved in the task are highly concrete. The model itself, and the furniture in the model, are tangible, three-dimensional objects. Each one is both a real object and a symbolic representation of something other than itself.

Second, successful performance requires that the child comprehend and exploit a symbolic relation—the relation between the model and the room. To solve the task, the child must understand that the location of the toy in the model specifies the location in the room. The concreteness of the model is useful to children only if it helps them understand the abstract stands-for relation between the model and the room.

Third, children are required to solve a seemingly familiar task (searching for a hidden toy) in a novel way. Typically, when young children search for hidden objects, they rely exclusively on direct experience; like adults, they often search where they have last seen an object. To solve our task, how-

ever, children have to adopt a totally new strategy that involves relying exclusively on information from the symbol.

Two sets of results from our research on children's use of scale models are very relevant to understanding the effects of concreteness on cognitive development. First, young children's understanding of the model is quite fragile. Children have trouble initially understanding the relation between the model and the room, and even after they do, they can easily lose sight of this relation. Second, the concreteness of the model may actually contribute to the fragility of children's understanding of the model-room relation. The concrete nature of the model may even make it *more* difficult for young children to use it as a symbol than a less concrete object, such as a photograph. In the next two sections we review both aspects of children's understanding of scale models.

The Fragility of Children's Understanding of Scale Models

Despite the apparent simplicity of the model, very young children have great difficulty using it. These results are summarized in Figure 8.1. Children younger than 3 years of age usually perform very poorly (only about 20% correct retrievals). The difficulty that children encounter cannot be attributed to forgetting the location of the toy that they observed being hidden. Almost all children succeed on the memory-based search in which they return to the model to retrieve the miniature toy. Thus, 2½-year-olds can remember the location of the toy in the model, but they tend not to use this knowledge to find the toy in the room. Figure 8.1 also reveals that most 3-year-old children succeed in the standard model task (averaging over 85% correct searches).

The success of the 3-year-olds whose performance is shown in Figure 8.1 is not, however, the end of the developmental story. Although 3-year-olds can solve the standard model task, they have great difficulty even if seemingly minor changes are made in the procedures. For example, DeLoache, Kolstad, and Anderson (1991) found that young children's performance depends very much on the physical similarity between the model and the room. When the furniture in the model and the room are extremely similar in appearance, 3-year-olds are very successful. However, if the objects in

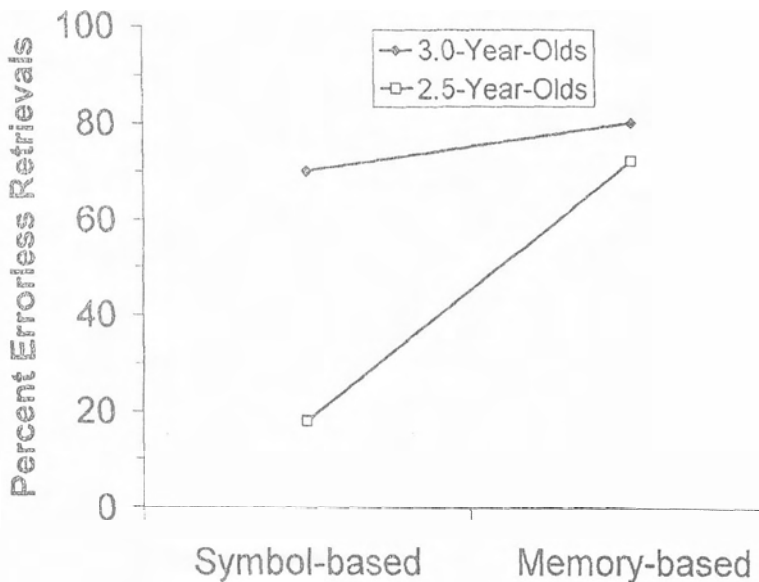


Figure 8.1 Children's performance in the original model study. Adapted from "Rapid Change in the Symbolic Functioning of Very Young Children," by J. S. DeLoache, 1987, *Science*, 238. For the symbol-based retrieval, children saw the miniature toy hidden in the model and then searched for the corresponding larger toy in the room. For the memory-based retrieval, children returned to the model and searched for the miniature toy. Note that only the symbol-based retrieval requires that children use the relation between the model and the room to find the toy.

the two spaces are dissimilar, the children perform at chance levels. Similarly, if the furniture in the model and in the room do not occupy the same relative spatial positions, performance deteriorates substantially (DeLoache, 1989).

Instructions are also critically important in children's comprehension and use of the model-scale relation (DeLoache, 1989). In the standard version of the task, we provide very specific and elaborate instructions about the correspondence between the model and the room. Providing less detailed instructions reduces 3- and even 3%-year-old children's performance to near-chance levels. It is not enough simply to tell the children that Little and Big Snoopy's rooms are alike and that the toys are hidden in the corresponding places in the two rooms. Instead, we must explicitly describe the relation and point out the correspondences between objects in the model and in the room (DeLoache, de Mendoza, & Anderson, 1999). Older children are less dependent on information from the expeller. Four-year-olds can succeed with the less detailed instructions described above, although they still need explicit information about the general model-room relation. Older children are more able to detect the relation on their own. A group of 5- to 7-year-old children were shown the model, the room, and the two toys. They then observed a hiding event in the model and were asked to find the larger toy in the room (with no explanation of the relations between the spaces or the hiding event.). Most of these older children inferred the "rules of the game" from this very minimal information and successfully retrieved the toy.

Even when children do initially grasp the relation between the model and the room, they may still have difficulty keeping track of the relevance of this relation for finding the toy. Uttal, Schreiber, and DeLoache (1995) showed that having to wait before using the information in the model to find the toy in the room caused 3-year-olds' performance to deteriorate dramatically. The task began as it usually does, with children watching us hide the toy in the model and then attempting to find the corresponding toy in the room. There was, however, one difference: We inserted delays between when the children saw the toy being hidden in the model and when they searched in the room. The delays were of three different lengths: 20 seconds, 2 minutes, and 5 minutes. Across the six search trials, all children experienced each of the delays twice. Different groups of children received the delays in one of three different orders. The groups were labeled in terms of the delay that they experienced first: the short-delay-first group had a 20-second delay first, the medium-delay-first group had a 2-minute delay first, and the long-delay-first group had a 5-minute delay first. After the initial trial, the children in each group received trials at the other delays, with delay length counter-balanced over trials.

As shown in Figure 8.2, the length of the initial delay greatly affected children's performance. The long-delay-first group performed poorly on all trials, but the short-delay-first group performed well on most of the trials. We can rule out one possible explanation for the poor performance of the long-delay first group: that children could not remember the location of the toy in the model during the initial delay. If this were true, then the children should perform much better on the shorter delay trials that followed the initial long delay. But this did not occur; the long-delay-first group performed generally poorly on all subsequent trials, even those trials with the short (20-second) delay that normally would give them little, if any, problem. Moreover, children could find the toy in the model even after the long delays. Thus they did not forget where the toy was hidden in the model; they instead forgot that the model could help them find the toy in the room. Uttal et al. (1995) concluded that, during the initial long delay, the children in the long-delay-first group lost track of the relation between the model and the room. Consequently, when they entered the room to search for the toy, they did not use the location of the toy in the model as a guide for searching in the room. The initial delay disrupted their tenuous grasp on the relation between the model and the room. Once the knowledge that the model could help was lost, the children continued to perform poorly, even on the subsequent, shorter delay trials.

Concreteness and the Dual Representation Hypothesis

What accounts for the fragility of young children's comprehension of the relation between the room and the model? In several studies we have demonstrated that the concreteness of the model is actually

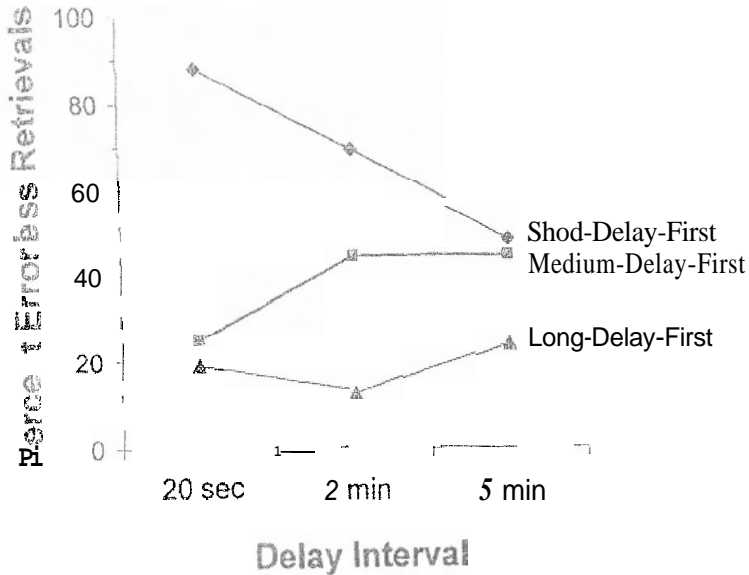


Figure 8.2 The effect of delay on children's use of a model. The initial delay led to much worse performance, even on the subsequent shorter days.

a cause of children's difficulty in using it as a symbol. Highly attractive and salient objects may be particularly difficult for children to think of as representations of something else—as symbols. This interpretation highlights the dual nature of the influence of concreteness on children's performance. Although concreteness may help children to perceive physical similarities between symbols and their referents, it also may make it more difficult for them to think about the abstract symbolic relation between the two.

A scale model such as the one used in our task has a dual nature; it is both a symbol and an object (or a set of objects) with a very high degree of physical salience. The very features that make it highly interesting and attractive to young children as a concrete object to play with, can obscure its role as a representation of something else. To use a model as a symbol, children must achieve *dual representation* (DeLoache, 1991, 1995, 2000; DeLoache & Burns, 1994; DeLoache, Miller, & Rosengren, 1997). They must mentally represent the model itself as an object and, at the same time, as a symbol for what it represents. In the model task, the child must form a meaningful mental representation of the model as a miniature room in which toys can be hidden and found, and he or she has to interact physically with it. At the same time, the child must represent the model as a term in an abstract, "stands for" relation, and he or she must use that relation as a basis for drawing inferences.

According to the dual representation hypothesis, the more salient a symbol is as a concrete object, the more difficult it is to appreciate its sole as a symbol for something other than itself. Thus, the more young children are attracted to a model as an interesting object, the more difficult it will be for them to detect its relation to the room it stands for.

The dual representation hypothesis has generated several interesting predictions. For example, it suggests that factors that decrease children's attention to the model as an interesting object should increase their use of the model as a symbol. In one study, 2-year-old children's access to the model was decreased by placing it behind a window (DeLoache, 2000). The children could still see the location of the toy in the model, but they could have no direct contact with the model. This manipulation led to better performance. Conversely, factors that increase children's attention to the model as an object should lead to a decrease in their use of the model as a representation of something else. This prediction also was confirmed. Allowing 3-year-old children to play with the model for 5 to 10 minutes led to a decrease in performance when children were asked to use it to find the toy in the room (DeLoache, 2000).

Another finding that supports the dual representation hypothesis concerns children's use of photographs, rather than the model, to find the toy. Two-and-one-half-year-olds, who typically perform very poorly in the standard model task, perform much better when a photograph is substituted for the model (DeLoache & Burns, 1993, 1994; DeLoache, Pierroutsakos, & Uttal, 2003). A photograph is less salient as an object and hence could be considered less concrete than a model is. Most obviously, the model is a three-dimensional representation, whereas the photograph is only two-dimensional. In support of the hypothesis, the 2½-year-olds performed much better with a photograph than their age-mates did with the model. In sum, the results indicate that a more concrete object, a model, may be more difficult to use than a less concrete object, a photograph.

DeLoache, Miller, and Rosengren (1997) have provided especially strong support for the dual representation hypothesis. In this research, 2½-year-old children were led to believe that a shrinking machine could shrink (and, subsequently, enlarge) a room. The idea was that if children believe that a scale model actually is a room that has been shrunk by a machine, then there is no symbolic relation between the two spaces; to the child, the model simply is the room. Hence, dual representation is not required, so children should have no trouble reasoning about the relation between the two spaces.

Each child was first given a demonstration in which a "shrinking machine" (an oscilloscope accompanied by computer-generated sounds described as the "sounds the machine makes while it's working") apparently caused a troll doll to turn into a miniature version of itself. The machine then supposedly "enlarged" the troll back to its original size. Next, the machine seemed to cause the "troll's room" (a tent-like room used in many previous model studies) to turn into a scale model identical to it except for size. It then enlarged the room.

The child then watched as the experimenter hid the larger troll somewhere in the portable room. After waiting while the machine "shrunk" the room, the child was asked to find the hidden toy. (The miniature troll was, of course, hidden in the same place in the model as the larger troll was in the room.) Thus, just as in the standard model task, the child had to use his or her knowledge of where the toy was hidden in one space to figure out where to search in the other. Unlike the standard task, there was no representational relation between the two spaces. As predicted on the basis of the dual representation hypothesis, performance was significantly better in this nonsymbolic task than in the standard model task. We know of no basis other than dual representation to explain this result.

The discussion thus far reveals that although the model is a highly salient concrete object, young children have difficulty using it as a symbol. Moreover, the concreteness of the model may be part of the problem, as children must look past the model's salient, concrete properties to understand that it is intended to be a representation of something else. These results have important implications for children's understanding of other symbol systems, such as letters and numbers. We explore these implications in the next section.

CONCRETENESS, DUAL REPRESENTATION, AND EDUCATIONAL SYMBOLS

In the preschool and early elementary school years, children are asked to master a variety of symbol systems, such as letters, numbers, maps, and musical notation. Symbolic reasoning is thus fundamentally important for educational achievement, and children who fail to become skilled in even one of the major symbol systems are at serious risk of being left behind.

The difficulty that children sometimes have in acquiring an understanding of these important symbol systems has led to a variety of materials that are designed to help children learn the relevant information. For example, teachers often use concrete objects as substitutes for abstract symbolic representations. These objects are often referred to as *manipulatives*. Examples of concrete, three-dimensional objects include Dienes Blocks, Base 10 blocks, Digi-Blocks, and Cuisenaire Rods. In addition, teachers use many household objects as informal manipulatives, including cereal, money, and paper clips (Stevenson & Stigler, 1992). Outside the classroom, parents can purchase a vast array of attractive objects of a symbolic nature in the hope that such objects will help their children acquire

early literacy and number skills. Magnetic letters and numbers cover a large proportion of the refrigerators in the homes of young American children, stuck there to encourage early learning.

Manipulatives have been touted as solutions for children of a wide range of ages and ability levels; they have been offered as appropriate for all ability levels, ranging from the disabled to the gifted (Clements & McMillen, 1996; Sowell, 1989; Wearne & Hiebert, 1988). Indeed, faith in the value of manipulatives is almost a defining characteristic of modern approaches to early childhood education. Unfortunately, however, research on the effectiveness of manipulatives has not confirmed the anticipated benefits. Several studies have shown, at best, inconsistent or weak advantages for manipulatives in comparison to more traditional techniques for teaching mathematics to children (Ball, 1992; Clements, 1997; Clements & McMillen, 1996; Hughes, 1986). Longitudinal and intensive studies of the use of manipulatives in classrooms have shown that children often fail to establish connections between manipulatives and the information that the manipulatives are intended to communicate (Sarama & Clements, 2002, 2004; Sowell, 1989). Put simply although manipulatives can facilitate thinking, they are not a panacea.

We suggest that part of the reason that manipulatives have not been shown to support symbol-based solutions involves challenges that are very similar to those that younger children encounter when using a scale model. There are at least two general similarities between what is required to succeed in our model task and what is required to effectively use a manipulative. The first is that the relation between a manipulative and what it is intended to represent may not be transparent to young children. In other words, the concreteness of a manipulative (or of our model) does not guarantee that children will understand that it is intended to represent something other than itself. To a teacher or parent, the relation between a manipulative-based solution and a more traditional written solution may seem obvious or even transparent. But the same may not be true in the minds of young children. As we discuss, the relation between manipulatives and other types of representations may be opaque to young children.

The second similarity between children's difficulties with our model and with manipulatives is that dual representation is relevant to both. As was true of our model, manipulatives have a dual nature; they are intended to be used as representations of something else, but they also are objects in their own right. In the next section, we review some difficulties that children encounter when using manipulatives, difficulties that parallel younger children's problems with our scale model and that are consistent with the dual representation perspective.

Children Often Fail to Grasp the Relation Between Manipulatives and Written Representations

From a teacher's point of view, the goal of using a manipulative is to provide support for learning more general mathematical concepts. However, this is no guarantee that children will see the manipulative in this way. Previous work on the use of manipulatives has documented numerous examples of mismatches between teachers' expectations and students' understandings. Even when young children do learn to perform mathematical operations using manipulatives, their knowledge of the two ways of solving the problems may remain encapsulated; that is, children often fail to see the relation between solving mathematics problems via manipulatives and solving the same or similar problems via abstract symbols (Uttal, Scudder, & DeLoache, 1997). For example, children may not see that the solutions to two-digit subtraction problems that they derive from manipulatives are also relevant to similar but written versions of the same problems. In the child's mind, the task of doing manipulatives-based arithmetic may be completely separate from doing written arithmetic.

An analogy to our scale model illustrates the differences between how students and teachers may view the relation between manipulatives and written representations of numbers. Our model is extremely concrete, and parents are amazed when the task proves difficult for intelligent, interested children. We believe that similar issues may arise when older children are asked to use manipulatives; to the teacher, the relation between the manipulative and a more abstract concept may be direct and

obvious, but this relation may be, and may remain, obscure to young children, particularly if the relation is not pointed out explicitly.

Evidence that children often fail to draw connections between manipulatives and more traditional forms of mathematical symbols comes from Resnick and Omanson's (1987) intensive studies of children's use of manipulatives and their understanding of mathematical concepts. Resnick and Omanson systematically evaluated third-grade children's ability to solve problems both with and without manipulatives. Much of the work involved Dienes Blocks, which are a systematic set of manipulatives that are designed to help children acquire understanding of base 10 concepts. Most of the children understood what was asked of them and appeared to enjoy working with the blocks. Unfortunately, however, the children's ease with and knowledge of the blocks was not related to their understanding of similar kinds of problems expressed in more formal mathematical terms. The children did not relate approaches they had used to solve problems with manipulatives to the solution of similar problems involving written symbols. For example, children who were successful in using Dienes Blocks to solve subtraction problems involving two or three digits had trouble solving simpler written problems. Indeed, the child who performed best with the Dienes Blocks performed worst on the standard problems. Clearly, success with a manipulative did not guarantee success with written symbols; in fact, success with one form of mathematical expression was unrelated to success with the other.

Other researchers have provided additional evidence of the nonequivalence of concrete and more abstract forms of mathematical expressions. For example, Hughes (1986) investigated young elementary school children's ability to use simple blocks or bricks to solve addition and subtraction problems. What is most interesting about this study for the current discussion is that the children were explicitly asked to draw connections between solutions involving concrete objects and those involving more abstract, written problems. The children were asked to use the bricks to represent the underlying concepts that were expressed in the written problems. For example, the children were asked to use bricks to solve written problems, such as $1 + 7 = ?$. The experimenter and the teachers expected that the children would use the bricks to show how the two numbers could be combined. For example, children might be expected to show 1 brick and a pile of 7 bricks. The process of addition could be represented by combining the single brick and the pile of 7 bricks to form one pile with 8 bricks. But this is not what happened. Overall, the children performed poorly. Regardless of whether they could solve the written problems, they had difficulty representing the problems with the bricks. Moreover, the children's errors demonstrated that they failed to appreciate that the bricks and written symbols were two alternate forms of mathematical expression. Many children took the instructions literally, using the bricks to physically spell out the written problems (Figure 8.3). For example, they

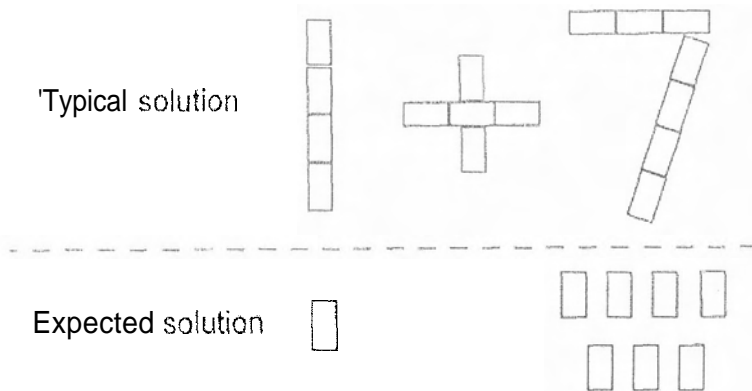


Figure 8.3 An example of how children use small bricks to represent the problem $1 + 7 = 8$. The children often copied written problems with the bricks rather than using the bricks as an alternate representational system. From *Children and Numbers: Difficulties in Learning Mathematics*, by M. Hughes, 1986, pp 99–103. Copyright 1986 by Basil Blackwell. Adapted with permission.

made a line of bricks to represent the “1” and two intersecting lines to represent the “+” and so on. These results again demonstrate that children may treat solutions involving manipulatives and those involving written mathematical symbols as cognitively distinct entities.

The research on children's understanding of inanimatives also highlights the conditions under which manipulatives are likely to be effective. Specifically, the results of several studies suggest that manipulatives are most effective when they are used to augment, rather than to substitute for, instructions involving written symbols. In successful cases of manipulative use, teachers have drawn specific connections between children's use of a manipulative and the related expression of the underlying concept in written form. For example, consider Wearne and Hiebert's (1988) program. It focuses on fractions, but the results are relevant to other mathematical concepts. At all stages of the program, the teacher draws specific links between manipulatives and written symbolic expressions. The manipulative is used as a bridge to the written expression; rather than as a substitute or precursor for written symbols. As a result, a scaffold is provided to assist children in learning written representations. The program gradually leads them away from a focus on concrete manipulatives and toward a focus on written representations. Thus, the focus of this and similar successful programs is on the relation between manipulatives and other forms of mathematical expression. Similarly, the *Building Blocks* curriculum (Sarama & Clements, 2002, 2004) uses manipulatives to help children gain insight into mathematical concepts, but it also includes activities to link manipulatives to other forms of representation. There is extensive use of concrete inanimatives, but the activities with the manipulatives are designed with the end goal of facilitating children's understanding of written representations. What makes this curriculum special, if not unique, is that there is, by design, a systematic formulation by which children grow out of using manipulatives. The materials are progressively layered, meaning that activities at earlier levels are designed to lay the foundation for later activities. In this way, the curriculum establishes linkages, both implicitly and explicitly, between manipulative-based solutions and written solutions.

Attractive Objects May Be Distracting Manipulatives

Another implication of the present analysis is that objects that are interesting in their own right may not make the best manipulatives. Observations of manipulative use in other countries have supported the idea that a good manipulative is not necessarily an inherently interesting object. For example, in Japan, children use the same set of manipulatives throughout the early elementary school years. Stevenson and Stigler (1992, pp. 186–187) who have conducted several cross-national comparisons of mathematics achievement in Asia and the United States, have observed the following:

Japanese teachers ... use the items in the math set repeatedly throughout the elementary school years. ... American teachers seek variety. They may use Popsicle sticks in one lesson, and marbles, Cheerios, M&Ms, checkers, poker chips, or plastic animals in another. The American view is that objects should be varied in order to maintain children's interest. The Asian view is that using a variety of representational materials may confuse children, and thereby make it more difficult for them to use the objects for the representation and solution of mathematics problems. Multiplication is easier to understand when the same tiles are used as were used when the children learned to add.

In summary, one of the challenges of effective use of manipulatives is that children sometimes have difficulty linking manipulatives-based solutions to written solutions. In this regard, the concreteness of the manipulatives may contribute to the problem by focusing children's attention on the characteristics of the objects themselves rather than on what the objects are intended to represent. It is important to stress that this perspective does not mean that manipulatives are not useful or are harmful to children's learning. Using manipulatives can indeed help mathematical thinking in several ways (see, for example, Martin & Schwartz, in press). Thus we do not deny that manipulatives can serve an important role in preschool and early elementary school education. However, effective use of manipulatives requires that teachers consider both the advantages and disadvantages of using manipulatives. In this regard,

we have identified a possible challenge of using manipulatives—children may have difficulty relating manipulatives-based solutions to written solutions.

Letters As a Symbol System

The questions raised in this chapter regarding children's acquisition of symbols also are relevant to the early development of reading. In learning to read, children must master the relation between an abstract symbol system and its referents. Given the importance of the alphabet and the problems children may have in learning it, parents often turn to other means of making letter learning more concrete. For example, concrete objects such as alphabet blocks or magnetic letters potentially can provide a tactile means of teaching reading in much the way that mathematics manipulatives allow hands-on learning of mathematics. Like mathematics manipulatives, concrete letters transform the abstractness of graphemes and phonemes into familiar, perceptually rich objects. Although the use of manipulatives for reading instruction has not been investigated as the use of manipulatives in math education has been, it seems likely that similar caution is appropriate. Simply putting the letters of the alphabet on magnets or on other toys does not guarantee that children will learn to use them for reading and writing rather than as building blocks. In this section we briefly review what children must learn to understand letter-sound correspondences and consider the possible influences of using concrete objects on this process.

Understanding letters is difficult because letters are noniconic symbols (Bialystok & Martin, 2003; Tolchinsky, 2003; Treiman, 2000). Unlike pictographs, there is nothing inherent in the structure of letters that reflects what they represent. In essence, understanding letters as notational symbols requires that children appreciate nonanalogous, noniconic symbolic relations (Bialystok, 1992; Munn, 1998).

Bialystok (Bialystok, 1992; Bialystok & Martin, 2003; Bialystok, Shenfield, & Codd, 2000) proposed that children must relinquish their hold on the specific perceptual properties of objects to understand them as symbols. Symbol acquisition emerges in three stages as children's initially fragile understanding of symbols becomes more flexible. Children first learn a set of symbols without understanding their relation to what they represent. For example, they may first be capable of verbally reproducing a sequence of symbols (e.g., counting in a series or reciting the alphabet). They may then begin to observe the relation of these objects to their referents. In this second stage, children tend to assume that the relations between symbols and referents are iconic and analogous. For example, they may believe that the word "ant" is shorter than the word "elephant" because ants are smaller than elephants. Similarly, Spanish and Italian children associate bigger words with bigger objects, in spite of the fact that this relationship is even less perfect in both of these languages. Both Spanish and Italian use suffixes to demarcate diminutives of root words, so that longer words actually denote smaller objects (Ferreiro, 1985). For example, the suffix "ita" in Spanish indicates the diminutive. When children finally acquire full symbolic competence in Bialystok's third stage, they are capable of understanding that symbols may be noniconic and nonanalogous (e.g., "car" is shorter than "banana," even though cars are larger than bananas). Thus, Bialystok has demonstrated that the acquisition of symbols such as letters and numbers occurs in a gradual three-step process, not as an abrupt concrete-to-abstract shift.

Once children know the correspondences between the written forms (graphemes) and auditory forms (phonemes), they have the requisite knowledge to read and write any word in the language (Ravid & Tolchinsky, 2002; Tolchinsky & Teberosky, 1998). Learning individual grapheme-phoneme correspondences, however, is neither easy nor a guarantee that children will learn to read. In fact, several studies (Landsmann & Karmiloff-Smith, 1992; Tolchinsky, 2003; Tolchinsky-Landsmann & Levin, 1985) have found that children's understanding of letters as part of a notational symbol system does not necessarily co-occur with their understanding of how the letters are used in referential communication. For example, Landsmann and Karmiloff-Smith (1992) asked children of ages 4 through 6 to invent nonletters, nonnumbers, and nonwords. Children in all age groups imposed different constraints on what qualified as nonletters and nonnumbers, demonstrating their understanding that

letters and numbers were separate domains of symbols but also that they are not in the same domain as drawings. For example, one child produced "tttt" when asked to generate a nonword. Only the older children, however, understood that symbols serve a referential role as well as a notational role. Rather than simply using strings of repeated letters to create nonwords, 5- and 6-year-olds generated nonwords that were unpronounceable and, thus, could serve no referential function.

These results reveal some of the challenges that young children face in learning to understand the symbolic properties of letters. Will making letters concrete facilitate children's understanding? According to the dual representation hypothesis, attempts to make alphabet blocks colorful and engaging as objects might detract the child from seeing the letters on them as symbols. The physical features or concreteness of the blocks actually may obfuscate the symbol-referent relation. Alphabet blocks, for example, typically are constructed in different colors, which facilitate children's perceptual differentiation of different letters when they are learning the alphabet early on. The elaboration of individual letters is similarly evident in the topical organization of *Sesame Street*, which typically focuses on only two letters of the alphabet per episode (i.e., "This episode brought to you by the letter 'E'") and in different skits that are used to interest children in learning their letters (e.g., the letter beauty pageant). Such attempts to make individual letters interesting may distract from the collective function the letters serve within the notational system as a whole. Emphasizing letters as perceptually salient objects in their own right may, in fact, make it more difficult to see each letter as being a component of a word and as serving an equivalent notational role in the alphabet.

RESEARCH ON THE EFFECTS OF PLAYING WITH CONCRETE OBJECTS ON CHILDREN'S UNDERSTANDING OF THE SYMBOLIC PROPERTIES OF LETTERS AND NUMBERS.

The discussion thus far of the influences of playing with concrete objects on children's understanding of educational symbols has been theoretical. We have suggested that there are direct links between our earlier research on concrete models and the challenges that children face in coming to understand educational symbols. We are now putting these ideas to the test. In recent research supported by the U.S. Department of Education, we have specifically investigated how playing with concrete objects affects children's understanding of the symbolic properties of letters and numbers. This is the first study to address directly how interaction with concrete objects affects children's understanding of how letters and numbers can be used as symbolic representations.

Separate studies were conducted to investigate children's understanding of letters and numbers, and we therefore refer to the studies as the *letter* and *number* studies. In both studies, we tested children who were just turning 4 (M age = 47.4 months) and children approximately 6 months older (M age = 53.5 months). The research took place in the children's homes. The researchers visited the children's homes three times to administer assessments or to demonstrate activities that children and parents could perform with traditional objects or concrete letters or numbers.

The first assessment provided baseline information regarding children's knowledge of letters or numbers and was based in part on subtests of the Woodcock-Johnson tests of preschool achievement or the Test of Early Mathematics Ability (TEMA) (Ginsburg & Baroody, 1990). We also included several measures of children's understanding of the symbolic properties of letters and numbers. The most important, the box labeling task, was adapted from Hughes (1986). It required that children use letters or numbers to keep track of the contents of three metal tins. In the letter study, we placed different toy animals (bear, duck, and frog) in the tin boxes; in the number study, we placed different quantities of paper "cookies" in the boxes. In both cases, the children were asked to "make something that will help [them] to remember what is in the box." This task gives children the opportunity to construct a symbolic representation to facilitate memory. The nature and quality of the representations that they construct can shed light on their conceptions of the process of representation and the symbolic properties of letters and numbers (see DeLoache, Simcock, & Marzolf, 2004; Eskritt & Lee, 2002; Hughes, 1986; Marzolf & DeLoache, 1994; Munn, 1998).

Importantly, we asked children to complete the tasks twice, once with crayons and once with magnet letters or numbers. Half the children in both the letter and number studies completed the task with the magnets first; the remainder completed the task with the crayons first.

At the end of the first testing session, we left a set of toys for the children to play with in the days between the sessions, and we demonstrated the games that children and parents could play. Children in the control group were assigned randomly to play with traditional toys and objects. For example, they blew bubbles with a bubble wand, they made simple jewelry with beads, and they played a simple basketball game with a suction-cup hoop and a sponge ball. Children in the experimental group were asked to play similar games but to use toy letters or numbers as the toys. For example, these children blew bubbles with letters (e.g., o and e) or with numbers (e.g., 6, 8, or 9). Likewise, they played basketball with letters and numbers. The parents were asked to encourage their children to play the different games, to keep a log of how often they played, and to take photographs of what they made with the toys or symbols, such as jewelry, towers, etc.

Approximately 5 to 7 days later, the experimenters returned to the children's homes. They did not administer tests or assessments at this second visit. Instead, they demonstrated a second set of games that the children could play during the next week and left the appropriate materials for these new games with the parents. The demonstrations helped to ensure that the children understood the games that we were asking them to play.

The researchers returned to the homes a third and final time a week later, to administer the final assessments, which included most of the tests the children had taken at the first session. By administering the tests both before and after children played with the toys or the symbolic objects, we were able to assess the effects of treating symbolic objects as toys on children's understanding of the symbolic properties and basic knowledge of letters and numbers.

For the most part, children's performance was not affected by the play activities; the children performed comparably, regardless of whether they played with toys or, with letters or numbers. Thus playing with the concrete objects neither helped nor hurt children's performance, either on the tests of symbolic knowledge or on the basic achievement tests (e.g., the Woodcock-Johnson preliteracy tests).

There was, however, an interesting effect of the type of objects (crayons or magnets) with which the children performed the box labeling tasks. First, in both studies, children performed better with the magnets than with the crayons. More specifically, they often placed the correct magnet on the box to represent the stuffed animal or the quantity of cookies contained within. In contrast, when using the crayons, the children were less likely to produce symbolic representations. For example, some of the children made drawings that seemed to have little discernible relation to the contents of the boxes. There was, however, evidence of transfer from using the magnets to using the crayons. Children who performed the box labeling task with the magnets first were more likely to use the crayon in a symbolic manner, such as to write a letter or number. Thus using the magnets not only helped the children perform better with the magnets; it also facilitated their performance with the crayon.

We believe that using the magnets in a symbolic fashion provided a basis for transfer to the crayon task. The children who performed the task first with the magnets were now more likely to approach the same task with the crayon as a form of symbolic representation. In other words, the magnets provided a scaffold that allowed children to use their nascent knowledge of symbolic relations. Four-year-olds possess some knowledge of the relation between letters and text, but they are unlikely to use this knowledge spontaneously (Bialystok, 1992; Bialystok & Martin, 2003). The magnetic letters or numbers gave the children the opportunity to use their knowledge of letters or numbers. Using the magnets as symbols then provided a basis for transfer to the more challenging crayon task.

These results are consistent with our theoretical perspective on the development of symbolic reasoning, particularly the dual-representation hypothesis. Simply playing with the concrete objects is not helpful, but using concrete objects in a symbolic way did improve performance. Several lines of research have already established that performing a symbolic task successfully can promote symbolic thinking in a more difficult domain. For example, DeLoache and colleagues showed that experience in

using a scale model helped children to use a symbol that they typically would not be able to use, a map (DeLoache et al., 2004; Marzolf & DeLoache, 1994). We believe that the magnetic letters or numbers provided a similar basis for transfer and thus helped children use the crayon in a symbolic manner.

In summary our results do provide evidence that using concrete objects can facilitate children's symbolic thinking. But it is very important to note that it is symbolic behavior with the concrete objects, and not simply playing with them, that provided the basis for the facilitative effect of the concrete objects. The concrete magnets facilitated children's symbolic thinking specifically because they helped the children to think about letters and numbers. Thus concrete objects can facilitate symbolic thinking; they are not a substitute for it.

IMPLICATIONS

The theoretical perspective that we have outlined above has important implications for the use of concrete objects in early childhood education. We have presented a theoretical perspective on the relation between concreteness and symbolic development that differs substantially from the traditional view. The ideas that we have developed in this chapter may prove useful in developing instructional strategies that maximize the effectiveness of manipulatives and other concrete objects.

Perhaps most importantly, our work reveals that *concreteness alone* does not convey *an inherent advantage*. Certainly there are circumstances in which working with concrete objects can in fact help children to acquire new knowledge or skills. However, this does not happen spontaneously simply because children work with a manipulative. For children to learn about symbolic relations from the use of concrete objects, the two forms of representation must be explicitly linked.

Importantly, our review also suggests that there are situations in which the *use of concrete objects may not be helpful and could even be harmful*. Like any learning technique or technology, concrete objects have both strengths and weaknesses. Highly attractive concrete objects may make it difficult for the child to think about using the concrete objects as representations of something else. Teachers and parents may want to reconsider the practice of providing children with highly attractive concrete symbols (e.g., letter magnets) and the expectation that this alone will facilitate symbolic development.

Finally, our review reveals that teachers must play a crucially important role in children's learning of symbolic relations. Whether a child draws a connection between a concrete object and a written representation depends critically on whether this relation is pointed out and reinforced by a teacher. Indeed, our review highlights specifically *how* teachers can integrate the use of concrete objects into instruction. Teachers can guide children's attention to the relation between manipulatives-based solutions and written representations of similar problems. The challenge for the teacher will be to decide specifically *how and when* such linkages should be made, but doing so should be an explicit goal of instruction.

CONCLUSIONS

Given the importance of learning symbol systems, it makes sense to try to help children in as many ways as possible. The use of concrete objects has been an important tool in this effort. Although we have raised serious questions about the use of concrete objects in early childhood education, we do not believe that the use of concrete objects should be eliminated or even reduced. Concrete objects such as letter blocks or number magnets can help children to discriminate one symbol from another and can awaken their interest in reading and mathematics. Moreover, our recent work has demonstrated that concrete objects can provide a scaffold on which an understanding of more abstract relations can be built.

Thus we would never endorse a proposal to eliminate the use of concrete objects in early childhood education. Our concern is not with the general use of such objects but rather with *how* they are used. The problems that we have cited only apply when the concrete objects are substituted for instruction or when the focus of children's activity is exclusively on highly attractive concrete objects. In such a

situation, children's attention centers on the objects themselves rather than on what the symbols are intended to represent. The desire to help children learn and to engage their interest by making objects interesting in their own right may at times be counterproductive. We advocate a balanced view, in which the disadvantages of using concrete objects are considered along with the advantages. Concrete objects are most useful when they are used to support or augment the learning of abstract concepts. They should not be used as substitutes for abstract representations.

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