CHAPTER 6

ON THE RELATION BETWEEN PLAY AND SYMBOLIC THOUGHT
The Case of Mathematics Manipulatives

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Most developmental psychologists and early childhood educators agree that young children learn best through play and exploration. As the chapters in this volume suggest, play and learning are intertwined for young children. Indeed, a focus on natural, play-based activities lies at the core of developmentally appropriate curricula. Organizations such as the National Association for the Education of Young Children (NAEYC) and the National Council for Teachers of Mathematics (NCTYM) stress that children's natural play should be the focus of preschool, kindergarten education, and (to a lesser extent) early elementary education (Uttal, Scudder, & DeLoache, 1997).

As used here, the term "play" does not mean only free play that lacks direction or purpose. Instead, I also use the term play to refer to structured

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activities in which teachers (or parents) guide children's activities. Teachers often plan activities and choose particular plaything with the goal of facilitating children's development or learning.

This chapter focuses on a quintessential example of this type of structured play, the use of mathematics ***manipulatives.*** Manipulatives are concrete objects (rods, blocks, etc.) that are designed to facilitate children's mathematical development. They are used extensively in early education. Their use is encouraged for children of all ability levels, including not only typical preschoolers but also gifted children and those with developmental disabilities (Ball, 1992; Uttal, Scudder, & DeLoache, 1997).

Manipulatives are constructed to allow children to learn naturally through play and exploration. There are formal manipulative systems, such as Dienes Blocks and Cuisenaire Rods, which are designed specifically to teach mathematics. However, teachers also use many informal types of manipulatives, which can include household objects (paper clips, coins, etc.) and pieces of candy or cereal. In addition, manipulatives have moved into the digital age. There are now several digital libraries of manipulatives, and computer scientists have created systems that combine the features of traditional hand-held manipulatives with advanced electronic technologies (Resnick et al., 1996, 1998). These "digital manipulatives indicate electronically when children have combined the manipulatives in the correct or expected **way.** For example, manipulatives representing the tens units in an addition problem may turn red and those representing the ones units may turn blue when the child places the objects in a particular order.

Obviously there are real and important differences in the types of manipulatives that young children are asked to use. It seems likely that different forms of rnanipulatives affect children's mathematical thinking in different ways (Chao, Stigler, & Woodward, 2000). However, our focus here is on a characteristic that most manipulative systems share; each is intended to represent mathematical information in a form that is tractable and does not require the use of written representations. It is this substitution for written representations, and the consequences of this substitution, that is the focus of this chapter. Accordingly, I have treated the term manipulatives in a general way, using it to refer to any physical system of objects that is (a) intended to help young children learn mathematical concepts, and (b) does not require that children use or comprehend written representations of the same concepts.

The fundamental assumption that motivates the enthusiasm regarding manipulatives is that young children understand mathematical concepts and relations in fundamentally different ways than older children's and adults' do. Young children's conception of mathematics is more concrete and tied to manipulations and transformations that can be performed manually. The theoretical basis for this belief is derived from the writings of Bruner (1966), Piaget (1964): and others. Each of these scholars

stressed, in different ways, the importance of concreteness and concrete objects on the development of children's concepts.

These theoretical backgrounds have been extended substantially in early childhood education, perhaps beyond the original intent of the theorists. Educators and developmental theorists alike have assumed that "concrete is inherently good; abstract is inherently not appropriate—at least at the beginning, at least for young learners." (Ball, 1992, p. 16) Manipulatives are thought to be particularly appropriate for young children because they encourage learning through natural exploration and play. On this view, manipulatives allow children to learn through play or at least in a playful manner (Ball, 1995; Uttal, Scudder, & DeLoache, 1997).

Despite the enthusiasm for the use of manipulatives, some researchers and teachers have raised questions about their efficacy. Both meta-analyses and intensive, longitudinal studies of children's mathematical development have not demonstrated that using manipulatives conveys a clear and consistent advantage in young children's learning of mathematics (Chao, Stigler, & Woodward, 2000; Hiebert, 1996; Sowell, 1989; Uttal, Scudder & DeLoache, 1987). In this chapter, I focus on one important difficulty that children seem to have that arises from the use of manipulative. Specifically, young children often fail to make a connection between concepts that they learn from manipulatives and written representations of the same or similar concepts. For example, a child might learn about the base-ten system through the use of Dienes Blocks, but the same child might gain no advantage in mastering the written representation of the base-ten system. Similarly, children might learn basic principles of addition from manipulatives but then fail to see a connection to the + sign (Bialystok, 1992; Resnick & Omanson, 1988; Fuson & Briars, 1991).

The difftculty that children sometimes have in connecting manipulatives with written, symbolic representations of the same problem is **a** fundamentally important one. At its core, mathematics is a formal symbol system, and learning written representations allows children to work on complex problems quickly and efficiently. Single symbols (the + sign, a function sign, etc.) can stand for a series of complex operations. Learning these symbols allows children to reason about relations independent of any physical instantiation of the concepts. For example, we can say, "What's **2 + 3**?" without thinking "**2** of what?" or "**3** of what?" **A** goal of mathematics education therefore should be the acquisition of a rich understanding of written symbols systems and of the ability to manipulate these symbols both on paper and mentally.

The symbolic demands of mathematics lead to an interesting paradox in regard to early education. On the one hand, children must acquire a symbol system that is distinctly not concrete; even simple mathematical symbols (such as + or -) bear no clear relation to their referents. Yet on the other hand, it is assumed that the best way to teach young children to understand mathematics concepts is through the use of concrete materials such as manipulatives. This paradox highlights the challenges and importance of helping children to establish linkages between manipulative-based solutions or representations and the corresponding written representations (Resnick & Omanson, 1988; Hiebert, 1986; Uttal, in press; Uttal, Liu, & DeLoache, 1999; Uttal, Scudder, & DeLoache, 1997).

The focus of this chapter is on both the opportunities and the difflculties that manipulative use engenders. I address the specific question of why children seem to have so much difficulty relating physical (concrete or manipulative) representations to written representations. I begin by documenting the problem, based on a review of several studies of classroom-based manipulative use. Next, I situate the problem of understanding the relation between manipulative-based representations and written representations within cognitive research on children's understanding of symbolic and representational relations. This literature review provides insights into the special challenges that children face when they are asked to relate one system of representation (e.g., manipulatives), to another (e.g., written representations). Finally, I conclude with specific recommendations regarding how teachers can best help children to understand and use manipulatives.

Two limitations of scope should be noted at the outset. First, the goal is not to provide a comprehensive review of research on the use of manipulative~.Instead, this chapter attempts to establish connections between research on manipulative use and other bodies of work in cognitive development. These linkages help to shed light on why young children may have difficulty relating manipulatives to written representations. Second, this chapter is intended to be neither an endorsement of manipulatives nor a critique of manipulatives per se. Manipulatives can be extremely effective, but like any instructional technique, they also have limitations and disadvantages. There are specific reasons why young children may have trouble linking manipulative mathematic solutions with written representations of the same problem. This information may prove very useful to teachers, curriculum designers, and parents who are interested in using manipulatives in an effective manner.

CHILDREN'S DIFFICULTIES IN RELATING MANIPULATIVES TO WRITTEN REPRESENTATIONS

The difficulty that children have in relating manipulatives to written representations is evident in many differerent contexts. The problem has been documented across a wide age range; it shows up in children as young as 4, but even high school students may have difficulty relating physical geometry constructions to written representations of the same problem (Von Glaserfeld, 1996). In this section I briefly review studies that have

demonstrated that children seem not to relate manipulatives-based representations to written representations of the same, or similar problems.

Resnick and Omanson (1988) conducted a particularly rich and detailed study of the acquisition of mathematical concepts from manipulatives and children's transfer (or lack of transfer) to written representations. Their study included a wide array of methods, ranging from intensive interviews of individual children to reaction time measures of children's processing of numerical information. Many of these measures were collected longitudinally, from the beginning to the end of the second grade. For these reasons, Resnick and Omanson's study provides a unique window onto children's manipulative-based learning and their (lack of) transfer of this knowledge to written skills.

The research documented that manipulative use can facilitate children's acquisition and fluid use of mathematics concepts. For example, children who regularly used Dienes Blocks acquired flexibility in subtraction skills such as borrowing. Many children who began the year with little or no knowledge of subtraction were able to perform well with the Dienes Blocks by the end of the year. In particular, many children's understanding of the borrowing procedures in subtraction, as evidenced by their Dienes Blocks constructions, increased substantially throughout the year.

However, there was very little correspondence between children's performance with the Dienes Blocks and their use of written representations of the same concepts. Indeed, the child who performed the best with the Dienes Blocks performed the *worst* when the testing involved written representations of what were essentially the same problems. And the opposite was also true: children who performed well with the written representations often had the most trouble using the Dienes Blocks.

Based on their results, Resnick and Omanson attempted to improve children's understanding by providing direct instruction about the relation between manipulative-based and written representations of subtraction facts. This was not an easy task; it took extensive, repeated instruction to help children grasp the relation between the two forms of representation. Many children did eventually appreciate the relation, but some persisted in treating the two systems as independent. Put simply, seeing the relation between the two systems of representation was a formidable challenge for these second graders; some never succeeded, and those that did succeed needed repeated, direct instruction about the relation between the two forms of representation.

Hughes (1986) documented a similar problem in elementary school children's use of concrete objects. He studied directly the relation between children's comprehension of manipulatives and their understanding of written representations of the same problems. In one task, children were asked to represent with manipulatives simple written addition problems, such as $1 + 7$. The children were given written problems and asked to show how the same problem could be represented with the manipulatives.

In general, children did not perform well; they had difficulty using the manipulatives to express written representations of the problems. The most striking examples illustrate children's difficulties in relating the two systems of representations. For example, some children simply *copied* the written problems with the manipulatives. They literally used the manipulatives to replicate the written problem. For example, the children would lay out the bricks to write "1 + 7 = ?" They saw the manipulatives as simply another way to write the problem; they used the bricks as if they were writing elements. The children either stuck with the manipulatives representation or with the written representation; they seemed to have difficulty construing the possibility of two *alternate* forms of representation. Hence they could not go back and forth between two forms of representation.

Children in this example had already acquired some understanding of written representations, and the problems that they faced were in some ways different from those illustrated in the Resnick and Omanson study. In Hughes' research, children had difficulty using manipulatives to represent written problems. In contrast, in Resnick and Omanson's study, children had difficulty moving from manipulatives to written representations. However, there is an important similarity between the two studies that will be explored further below: In both situations, the children had difficult dealing simultaneously with two, alternate forms of representation. They often could succeed with manipulatives or with written representations, but they failed to connect the two (See also Hiebert, 1989; Hiebert & Carpenter, 1992; Lesh, 1999).

THE DEVELOPMENT OF CHILDREN'S CONCEPTIONS OF SYMBOLS AND REPRESENTATIONS

The examples discussed in the previous section illustrate that children have trouble linking representations based on manipulatives with written, symbolic representations. In this section, I demonstrate that research on the development of children's understanding of symbolic representations is highly relevant to understanding the difficulty that children have in linking manipulatives to written representations. I review the results and implications of two lines of research on specific aspects of cognitive development in preschoolers and young elementary school children. The first concerns how children establish an initial insight into the relation between a symbol and its intended referent. The second concerns the development of the ability to reason systematically about relations between two alternate representations or construals of the same fact or concept. Taken together, these two programs of research highlight the likely sources of the difficulty that very young children experience in using manipulative~In addition, reviewing these lines of research leads directly to specific

solutions regarding how best to help children make connections between alternate forms of representations of mathematics concepts.

It should be noted that the relation of these research programs to manipulative use might not be immediately obvious. Neither research program has dealt specifically with manipulative use. Moreover, neither research program has focused on the development of children's mathematical concepts, and each program involves relatively short testing sessions. Nevertheless, these basic research programs shed light on the fundamental challenges of using symbols and of relating one form of representation to another. The research highlights critical aspects of children's thinking that ultimately are very relevant to the challenges that children face in using manipulatives.

Symbolic Development

Children's comprehension of symbols is obviously related to their understanding of mathematics. This is especially true given that much of the value of mathematics is gained from learning to manipulate a symbol system. Many of the challenges that children encounter in learning to understand symbolic relations are similar to those that they encounter in relating manipulatives to written representations.

Recent work in cognitive development has investigated the development of what is perhaps the core aspect of symbol use: understanding that one thing stands for another. A particularly relevant set of tasks involves the child's use of a novel symbol, a simple model, to find a hidden object (DeLoache, 1987; 1991; 2000; DeLoache, Miller & Rosengren, 1997; Uttal, Schreiber, & DeLoache, 1995). The child is asked to perform a familiar task, looking for a hidden object, in an unfamiliar way. The task is interesting and motivating to young children, who very much want to find the hidden object. These characteristics of the task allow researchers to gain a window onto the process by which children come to understand the basic relation between a symbol and its referent.

The task begins with an extensive orientation, during which the experimenter points out the correspondence between the model and the room. First, the experimenter points out the correspondence at a general level, referring to the model as "Little Snoopy's Room" and the room as "Big Snoopy's Room". Then, the experimenter points out correspondences between individual pieces of furniture in the model and in the room. For example, the experimenter demonstrates the relation between "Big Snoopy's Sofa" and "Little Snoopy's Sofa." Next, the experimenter hides the miniature toy, Little Snoopy, in the model and asks the child to find Big Snoopy in the room. The experimenter reminds the child that Big Snoopy is hiding the same place in his room that Little Snoopy is hiding in his room. The child is allowed to search until he or she finds the toy, but searches are scored as correct only if the child's first search is at the correct location.

This task has been conducted with children approximately ages 2 to 4. 2-½-year-olds typically fail, performing at chance levels. However, children only 6 months older perform dramatically better; 3.0-olds average approximately 75 % correct searches.

What accounts for the 2.5-year-olds' poor performance and for the dramatic improvement in 3.0-year-old$ performance? The younger children's failure is ***not*** due to memory. The task includes a memory check; after searching in the room, the child is asked to return to the model and find the miniature toy. If the child succeeds in this second retrieval (*Retrieval 2*), then memory for the location of the miniature toy cannot be the cause of the difficult finding the larger toy in the room. Almost all children succeed on Retrieval 2, regardless of their performance on Retrieval 1. Thus the children did know where the toy was hidden in the model but they could not use this knowledge to find the larger toy in the room that the model represented.

If memory is not the problem, then why do very young children have so much difficulty using the model as a symbol for the room? The answer lies in children's appreciation of symbolic relations. A specific challenge concerns an appreciation of what DeLoache and colleagues have termed ***dual representation.*** There are two ways to think of the model. The first is as an interesting object in its own right; the model contains, for example, several miniature pieces of furniture, a toy dog, etc. The second is as a representation of the room; the model is intended to stand for the room. To find the toy, the child must focus on one of these construals and not on the other. The children must think about the model as a symbolic representation rather than as an interesting object in its own right.

To an adult, these two interpretations of the model may seem almost inseparable; it is difficult to think of the model in isolation—to ignore that it is a representation of the room. In the mind of an adult, the purpose of the model is to represent the room. This interpretation of the purpose of the model would be difficult for an adult to put out of mind, particularly after the extensive orientation in which the experimenter pointed out the correspondence between the model and the room.

However, young children probably do not share with adults an understanding of the relation between the model and the room. Several lines of research suggest that the challenge for young children is to think of the model as a representation of the room, rather than as simply an interesting thing in it's own right. For example, manipulations that ***increase*** the salience of the model as a representation in its own right *decrease* the likelihood of children using the model as a symbol. This claim is based on research in which children were encouraged to play with the model before they were asked to use it as a symbol (DeLoache, 2000). When the child

arrived at the laboratory, the model was sitting in the middle of the room. Several toys, including the miniature dog, were placed in and around the model. The children were allowed to play with the model freely for 10 minutes. Thereafter, the experimental procedures were identical to the prior studies. Children in this group averaged only 41% correct searches, compared to more than 75% in the typical task in which children do not play with the model before they are asked to use it as a symbol.

Interestingly, the opposite is also true. Manipulations that ***decrease*** the salience of the intended symbol as an object in its own right ***increase*** children's success in establishing the symbolic correspondence. In this research, DeLoache and colleagues (2000) placed the model behind a pane of glass. Children could see the model but they could not touch it or otherwise interact with it. The experimenter pointed to the object that corresponded to the hiding location in the room. Two-and-a-half-year-olds, who typically fail the standard model task, performed much better when the model was placed behind glass. Placing the model out of reach made it impossible for the children to treat the object as a plaything and to focus on its properties as an object per se. Consequently, they were more able to focus on the model's relation to the room and hence they succeeded in the search task.

A fascinating line of research provides very strong evidence that the unique problem for young children involves using the model as a symbol. In this research (DeLoache, Miller, & Rosengren, 1997), 2.5-year-olds were made to believe the model zuas a shrunken version of the room rather than a symbolic representation of the room. The children were told that the experimenter had invented a shrinking machine that could shrink the room, the furniture, and the toys. The "room" in this study was actually a large tent-like structure composed of fabric suspended from PVC pipe. This "portable room" allowed the experimenter to easily disassemble the room and to replace it with a much smaller version during the shrinking procedure. Likewise, the small room (the model) could easily be replaced with the full size version during a "blowing up" trial.

The experiment began with a demonstration trial. The experimenter showed the child a full-size troll doll and said that a shrinking machine would now shrink the troll. The experimenter and the child left the room, but the child could hear strange sounds coming from the room; the child believed that these sounds were the shrinking machine in action. While the child and experimenter were out of the room, an assistant replaced the full-size troll with a miniature version. The experimenter and child then returned to the room. The experimenter pointed out the "success" of the shrinking machine.

Next, the experimenter introduced the test trials. She told the child that the troll would be hidden in the room and that the machine would then shrink (or blow up) the room, the troll, and the furniture. The experimenter hid the toy while the child watched. Tine experimenter and child then left the room, and the experimenter "activated the shrinking machine. Upon return, the child found that the room and its contents had been shrunken (or blown up). (In reality, a group of assistants had replaced the full size room with the miniature model). This basic procedure was repeated several times. On each trial, the experimenter either "blew up" the model to form the room or "shrunk the room to form the model. The child saw where the toy was hidden before the size change was simulated. Then, he or she had to find the toy in the shrunken (or blown up) version of the space.

The 2.5-year-old children performed well in the "shrunken room" task, even though they almost always failed the standard model task. This finding is particularly interesting when one considers that the two tasks are essentially the same: In both cases, the child must use the model (or small room) to find a toy that is hidden in the larger room. There is, however, one important change in terms of what the child thinks about the two spaces. In the standard task, the child needs to think of a symbolic, representational relation. In contrast, in the shrunken-room task, the child needs only to think about one room. This room is altered in size, but in the mind of the child it is the same room he or she saw before. In sum, the shrunken room task eliminates the need to think about symbolic relations, and consequently very young children succeed. These results provide very strong evidence that the challenge for young children is to think of the model as representing the room. Once this challenge is removed, children who normally fail do very well.

Seeing One Thing in Two Different Ways

A second, related line of research also sheds light on children's difficulty in understanding the relation between manipulatives and written mathematical symbols. This research program focuses on children's appreciation that a single stimulus or object can be interpreted in more than one way. Young children only gradually develop an ability to see one thing in two different ways. This ability may be critical to reasoning simultaneously about the relation between manipulatives and other forms of mathematics representation, particularly written symbols.

A classic demonstration of developmental differences in children's appreciation of multiple perspectives on the same stimulus concerns children's perception and understanding of ambiguous figures. These figures are well known in psychology. They include, for example, a figure that can be seen either as wrinkled old lady or a beautiful young lady. Similarly, another figure can be perceived either as a man or a mouse. A third ambiguous figure can be perceived either as a rabbit or a duck. Most adults have noticed that these figures can be perceived in both ways. For example, adults acknowledge that the figures could be interpreted either as an old or young women, either as a rabbit or a duck, etc. This does not mean that they can see both interpretations of the figure simultaneously. Instead, adults often report that the figure seems to switch back and forth from one construal to another.

Interestingly, children less than five or six do not seem to see or think about ambiguous figures in the same way as adults. Young children do not reverse ambiguous figures, even when prompted to do so. In one study, Gopnick and Rosati (2001) asked 3-, 4-, and 5-year-olds to look at an ambiguous figure and to describe what the figure looked like. The children were prompted to think of alternate interpretations of the figure. The researchers even went so far as to suggest the specific alternate interpretation of the figure. For example, if the child said that the figure looked like a bunny, the experimenter would ask if it could also be seen as a duck. The children were also asked to look at the figure for an additional minute to see if an alternate interpretation came to mind.

Most of the 3 and 4-year-olds interpreted the figures in only one way; even after prompting, they persisted with their original interpretation of the figures. Five-year-olds, however, often reported that the figures could be construed in more than one way; many spontaneously pointed out that the figure could be, for example, either a duck or a bunny. These children acknowledged the inherent ambiguity of the figures and said that other children might also see the figure in more than one way (See also Rock, Gopnick, and Hall, 1994).

Other lines of work point to the generality of these findings; young children's difficulty in appreciating multiple perspectives or interpretations is not limited solely to ambiguous figures. For example, Taylor, Cartwright, & Bowden (1991) investigated 4 and &year-olds understanding of ambiguity in drawings. Specifically, they showed children portions of drawings and asked them to identify the represented object. Because the researchers initially showed the child only a small portion of the drawing, the interpretation was often ambiguous. For example, the researcher would show the child a triangle, which was part of a large drawing of a witch; the triangle represented the witch's hat. Then the experimenter would expose the entire figure and ask the child what he or she saw. Almost all children said, "A witch". Next, the experimenter described a hypothetical task in which another child was asked to look at the triangle portion of the figure, with the remainder covered up. The 4-year-olds said that the new child would see a witch, even though only a triangle was visible. The children failed to appreciate that the figure could be seen in two different ways, and that which version one saw depended upon how much information was exposed. Once the children had seen the disambiguating information (e.g., the entire witch), they seemed unable to think simultaneously about the other interpretation (the triangle that formed the witch's hat (See also Chandler & Sokol, 1999; Sodian, 1990; Taylor, 1988).

These results have been interpreted as suggesting that children develop the ability to interpret the same stimulus in two different ways around age 5 or *6*. Before this age, children do not seem to reverse ambiguous figures or to appreciate the ambiguity that is inherent in many representations. Once they see something one way, they find it almost impossible to see the same thing in another way. **As** discussed below, this finding may have important implications for understanding the challenges that children face when working simultaneously with manipulatives and written, symbolic representations.

RELATION OF RESEARCH ON SYMBOLS USE AND REPRESENTATION TO MANIPULATIVES

There are at least three important similarities in the ***process*** of understanding symbolic relations and problems that children encounter in understanding relations between manipulatives to written representations (Uttal, DeLoache, & Scudder, 1997). First, the research reviewed in the prior section illustrates that achieving insight into a symbolic relation is not an easy or automatic process for young children. Children's understanding of symbolic relations is easily affected by many factors, and children quickly lose sight of the intended relation between a symbol and what it represents. Likewise, there is no guarantee that children will grasp the relation between a mathematics problem that uses manipulative and a similar problem that is expressed in writing. To a teacher, the correspondence between the two may seem obvious and even trivial, but to young children, the correspondence may remain opaque. Adults are experienced in using multiple symbol systems, but each new symbolic insight may be a challenge for young children.

Second, both research programs provide insight into why children may have trouble reasoning simultaneously about two different forms of representation, even if children understand written representations of mathematics problems. The research demonstrates that children tend to conceive of a stimulus or a concept in a single way, and that they do not spontaneously (and sometimes even with prompting) consider alternate construals of the same stimulus. Thus, when they are asked initially to reason about the relation between manipulatives and written representations, they may have substantial difficulty thinking about the two representational forms at the same time. Viewed from this perspective, Hughes' (1986) results also make sense; children sometimes copied the written representation with the manipulatives because they had difficulty thinking simultaneously about the two forms of mathematic representations at once. Just as children in Gopnick and Rosati's experiment failed to see that an ambiguous figure could be either a duck or a rabbit, children may fail

to see in their kindergarten classroom that a manipulative-based problem could also be interpreted in terms of a written representation. Tasks that require children less than 7 to think about the same stimulus or concept in two different ways may be inherently difficult.

Third, the research on symbolic development has a very strong implication regarding the central themes in this volume: Play may not be the best way to learn about symbolic relations. Recall that playing with the scale model actually decreased children's use of the model as a symbol. This example clearly illustrates that play may not be helpful when the educational task involves learning symbolic relations. Playing **with** an object that is intended to represent something else may increase children's attention to the properties of the object per se. Consequently, children may find it more difficult to focus on the object as a symbol. The same may well hold true for using manipulatives; playing with concrete objects may increase children's interest in mathematics but it may also make it more difficult for children to understand how the manipulatives relate to written representations. Therefore it is perhaps not surprising that the children in Resnick and Omanson's study who were most successful with the manipulatives were the least successful in using written representations.

IMPLICATIONS FOR INSTRUCTION WITH MANIPULATIVES

The previous discussion has several implications for understanding how to help young children to use manipulatives. In this final section, I consider implications for instruction involving manipulatives.

Manipulatives Cannot Be an End in Themselves

Perhaps the most general contribution of this chapter is to demonstrate that manipulatives cannot be used in isolation. Simply playing with **a** manipulative is unlikely to help children learn information that will facilitate their understanding of written representations. Indeed, it may sometimes be counterproductive.

However, this admonition does ***not*** mean that children do not learn from using manipulatives. Manipulatives can facilitate specific types of mathematical reasoning (Chao, Stigler, & Woodward, 2000; Resnick & Omanson, 1988). The problem is that this knowledge if often disconnected from other (written) representations. Manipulatives therefore can be only one part of an integrated system of instruction (Hiebert et al., 1997). The problem from the point of view of this chapter is not that manipulatives do not work; the problem is that they sometimes have been assumed to work in an almost magical fashion (Ball, 1992). Like all instructional techniques, rnanipulatives have their advantages and disadvantages.

Manipulatives Do Not Obviate Teachers.

The examples throughout this paper make one point especially clear: Whether manipulatives will help, hurt, or make no difference in children's acquisition of mathematics concepts depends greatly upon the role of the teacher in the process. Students must be guided to help make a discovery with the manipulative. Without such guidance, manipulatives may do as much harm as good.

More specifically, teachers play a critical role both in helping children understand how the manipulative system represents number and in linking those representations to written representations. The challenge for the teacher will be to figure out when, and how, to introduce and reinforce correspondences between the manipulative representation of a mathematics problem and it's corresponding written representation. Traditionally, it has often been assumed that manipulative use should precede the introduction of symbolic representations (Uttal, Scudder, & DeLoache, 1997). Proponents of this view suggest that children should first grasp the initial concepts through manipulatives, and that written representations should be introduced only after the child has fully grasped the concept that the manipulatives can represent. It may be useful, however, to consider introducing written representations at the same time as manipulatives-based representations, so that children do not segregate the two types of solutions. This suggestion would be most appropriate for children of ages 6 or older, who are more likely to appreciate that the same problem can be represented in more than one way.

Effective Manipulative Use Takes Time

Several lines of research have shown that for manipulatives to be effective, they must be used repeatedly for the same concept (Chao, Stigler, & Woodward, 2000; Fernandez, Clea; Yoshida, Makoto; Stigler, J.W, 1992; Hiebert & Carpenter, 1992; Hiebert & Wearne, 1993, 1996; Hiebert et al., 1997). Part of ;he reason is practice; children need time to learn how the manipulatives work, how different numbers and operations are represented, etc. However, research on symbolic development also points to another reason: over time children lose interest in the manipulatives as objects in themselves. Consequently, it may be easier for the children to

think about how the manipulatives relate to written representations. In other words, when rnanipulatives are first introduced, they are interesting as objects in themselves, and hence the potential relation to written representations may be difficult for children to perceive. However, with extensive practice, the manipulatives become a normal part of the classroom activities, and hence the students may now be able to focus on what the manipulatives are intended to represent in writing.

Attractive or Interesting Manipulatives May Not Always be Best

A related suggestion concerns the validity of a common assumption regarding the value of interesting or attractive manipulatives. It is often assumed that manipulatives should be interesting and attractive to be effective. However, the review of research on symbolic development strongly suggests that attractive manipulatives may sometimes be counterproductive; they may cause children to focus on the superficial properties of the manipulatives as objects rather than on their relation to written representations (See also Gentner & Ratterman, 1991).

In this regard, it is interesting to note that not all teachers emphasize diversity in choosing manipulatives. For example, Japanese teachers tend to stick with a limited set of manipulatives and use these consistently throughout the early elementary school years. Stevenson and Stigler (1992), who have conducted extensive research on cross-cultural differences in mathematics achievement, have observed the following:

> Japanese teachers... use the items in the math set repeatedly throughout the elementary school years... American teachers seek variety. They may use Popsicle sticks in one lesson, and marbles, Cheerios, M&M's, checkers, poker chips, or plastic animals in another. The American view is that using a variety of representational materials may confuse children, and thereby make it more difficult for them to use the objects for the representation and solution of mathematics problems (pp. 188187).

SUMMARY AND CONCLUSIONS

There is no doubt that play is critical to child development. But when learning involves an appreciation of symbolic relations, play may be a dual-edged sword. On the one hand, playing with an object may increase children's interest in and attention to the object. But on the other hand, playing with an object may cause children to have difficulty focusing on what the object is intended to represent. Put another way, the development of an understanding of symbolic relations requires that children *distance* themselves (Sigel, 1993) from the properties of the symbols as objects. Play may at times make distancing more difficult. In this chapter, I have applied this analysis to the use of manipulatives in early mathematics instruction. My goal in chis chapter has been to bring a new perspective to understanding both the advantages and disadvantages of using manipulatives to help children gain insight into mathematics concepts.

The general enthusiasm that many teachers have for manipulative use is well motivated. No one would want to return to the days in which the teaching of mathematics involved the repeated memorization of facts that were meaningless to young children. Manipulatives are developmentally appropriate in the sense that they are designed to match young children's level of understanding, which often focuses on concrete properties of objects and relations. But acquiring a mathematics concept from manipulative~does not guarantee that children will understand how or why the same concept can be expressed in writing. While it is certainly true that young children learn best through play, it is also equally true that teachers must place this play into an educational context. There is nothing magical about manipulatives; like any form of instruction, they have both strengths and weaknesses. Teachers have the critically important task of taking advantage of what manipulatives can offer while helping to prevent the problems that they can engender.

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