Gentner, D., Brem, S., Ferguson, R. W., Wolff, P., Markman, A. B., & Forbus, K. D. (1997). Analogy and creativity in the works of Johannes Kepler. In T. B. Ward, S. M. Smith, & J. Vaid (Eds.), *Creative thought: An investigation of conceptual structures and processes* (pp. 403-459). Washington, DC: American Psychological Association.

16

Analogy and Creativity in the Works of Johannes Kepler

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Analogy seems to have a share in all discoveries, but in some it has the lion's share.

(Polya, 1954, p. 17)

"The roads by which men arrive at their insights into celestial matters seem to me almost as worthy of wonder as those matters in themselves."

—Johannes Kepler

Analogy is often linked with creative thought (Finke, 1990, 1995; Finke, Ward & Smith, 1992; Gentner, 1982; Hesse, 1966; Holyoak & Thagard, 1995; Koestler, 1963; Perkins, 1994; Ward, Finke, & Smith, 1995). Boden (1994b) stated that "a psychological theory of creativity needs to explain how analogical thinking works" (p. 76). Our goal in this chapter is to illuminate the processes by which analogy promotes creativity and

This work was supported by National Science Foundation Grant BNS-87-20301 and Office of Naval Research Grant N00014-92-J-1098. Ron Ferguson received support from a Northwestern University Cognitive Science Fellowship.

conceptual change. We lay out four mechanisms by which analogy can act to create changes in knowledge, and consider the sorts of changes they promote.

We draw on the works of Johannes Kepler (1571–1630) to illustrate our points. Kepler is a particularly apt subject for studying analogy in discovery. He was a highly creative thinker, whose work spans and contributes to a period of immense change in scientific theory. He was also a prolific and intense analogizer. His writings teem with analogies, ranging from playful to serious, and from local comparisons to large extended analogies that evolved over decades and that were central in his discoveries.

We examine Kepler's use of analogies as revealed in his major works and his journals. Before doing so, however, we first address two important points. First, we want to be clear that in analyzing and simulating Kepler's analogies we are not claiming to be capturing anything like the whole of his thought processes. We are merely trying to be as explicit as we can, with the understanding that much is left to be explained. Second, we use Kepler's writings to infer his thought processes. To what extent is this justified? In particular, can we assume that his extended analogies were actually used in his thought processes, as opposed to being merely rhetorical devices? There are some grounds for optimism on this point, for Kepler's writings are unusually rich in descriptions of his thought processes. Many of Kepler's commentators have noted the exceptional—at times even excessive—candor and detail of his scientific writing. Holton (1973), in noting that Kepler has been relatively neglected among the great early scientists, stated

> [Modern scientists are] . . . taught to hide behind a rigorous structure the actual steps of discovery—those guesses, errors, and occasional strokes of good luck without which creative scientific work does not usually occur. But Kepler's embarrassing candor and intense emotional involvement force him to give us a detailed account of his tortuous process. . . . He gives us lengthy accounts of his failures, though sometimes they are tinged with ill-concealed pride in the difficulty of his task. With rich imagination he frequently finds

> analogies from every phase of life, exalted or commonplace. He is apt to interrupt his scientific thoughts, either with exhortations to the reader to follow a little longer through the almost unreadable account, or with trivial side issues and textual quibbling, or with personal anecdotes or delighted exclamations about some new geometrical relation, a numerological or musical analogy. (pp. 69–70)

Kepler's inclusiveness stemmed in part from his possibly overoptimistic rather naive belief that readers would wish to follow "the roads by which men arrive at their insights into celestial matters." In the introduction to the *Astronomia Nova* (Kepler, 1609/1992) he states this agenda:

> Here it is a question not only of leading the reader to an understanding of the subject matter in the easiest way, but also, chiefly, of the arguments, meanderings, or even chance occurrences by which I the author first came upon that understanding. Thus, in telling of Christopher Columbus, Magellan, and of the Portuguese, we do not simply ignore the errors by which the first opened up America, the second, the China Sea, and the last, the coast of Africa; rather, we would not wish them omitted, which would indeed be to deprive ourselves of an enormous pleasure in reading. (p. 78)

Kepler (1609/1992) was explicit in his intention to share the difficulties of discovery: "I therefore display these occasions [errors and meanderings] scrupulously, with, no doubt, some attendant difficulty for the reader. Nevertheless, that victory is sweeter that was born in danger, and the sun emerges from the clouds with redoubled splendour" (p. 95). Accordingly, Kepler frequently included long, tedious sections of calculations made in pursuit of false assumptions, informing the reader afterward that the line of reasoning had been wrong from the start. In the midst of one such section he wrote, "If this wearisome method has filled you with loathing, it should more properly fill you with compassion for me, as I have gone through it at least seventy times" (p. 256). This is not to say that Kepler's writings are pure diaries; his commentators note that some filtering and organizing took place. But his fascination with the cog-

nitive process of discovery led him to preserve much of the trail. A striking case occurred in 1621 when he published a second edition of his first book, the *Mysterium Cosmographicum* (Kepler, 1596/1981). Kepler's ideas had changed radically in the 25 intervening years, yet he chose not to rewrite but to leave the original text intact, adding notes that specified how and why his ideas had changed. He commented on why he preserved the errors in the original: "I enjoy recognizing them, because they tell me by what meanders, and by feeling along what walls through the darkness of ignorance, I have reached the shining gateway of truth" (Kepler, 1596/1981, p. 215).

Finally, Kepler includes a running commentary on his reactions. He makes the kinds of remarks that modern scientists cull from their papers: for example,

> If I had embarked upon this path a little more thoughtfully, I might have immediately arrived at the truth of the matter. But since I was blind from desire [to explain the deviation from a circular orbit] I did not pay attention to each and every part . . . and thus entered into new labyrinths, from which we will have to extract ourselves. (Kepler, 1609/1992, pp. 455–456)

or, from the same work, "Consider, thoughtful reader, and you will be transfixed by the force of the argument . . ." and again,

> And we, good reader, will not indulge in this splendid triumph for more than one small day . . . restrained as we are by the rumours of a new rebellion, lest the fabric of our achievement perish with excessive rejoicing. (p. 290)

The open spontaneity of Kepler's writing offers encouragement for the belief that his writings were at least partly reflective of this thinking.

TRACING KEPLER'S ANALOGIES

Kepler was a prolific analogizer. Not only in his books but also in his journals and letters, he used analogies constantly. In some cases the analogies seem simply playful. In other cases, analogizing is integral to his theorizing. This is consistent with research showing that analogies to prior knowledge can foster insight into new material (Bassok, 1990; Bassok & Holyoak, 1989; Catrambone & Holyoak, 1989; Clement, 1988; Dunbar, 1995; Forbus, Gentner, & Law, 1995; Gentner, 1982; Gentner & Gentner, 1983; Gentner, Rattermann, & Forbus, 1993; Gick & Holyoak, 1980, 1983; Holyoak, Junn, & Billman, 1984; Keane, 1988; Novick & Holyoak, 1991; Novick & Tversky, 1987; Ross, 1987; Spellman & Holyoak, 1993; Thagard, 1992).

Kepler returned to certain analogies repeatedly across different works, extending and analyzing them further on successive bouts. In this chapter, our goal is to characterize the mechanisms by which these analogies led to creative change in knowledge. We first briefly summarize the course of discovery that led him to his new account of celestial mechanics, including his use of extended analogies. We then trace Kepler's analogical processes, using structure-mapping theory to trace his inferences and analogical extensions. We show that with reasonable representational assumptions we can simulate some, though not all, of his mapping processes in a plausible manner.

KEPLER'S CELESTIAL PHYSICS

Johannes Kepler (1571–1630) is today best known for his three laws of planetary motion.[1] His far more important contributions in changing people's conception of the solar system are difficult to appreciate—ironically, in part because of his very success. The conceptual structure that existed prior to Kepler's work is now almost impossible to call forth. When Kepler began his work, the dominant view was that the heavenly bodies revolved around the Earth, supported by crystalline spheres, traveling at uniform speed in orbits made up of perfect circles. This Greek system, perfected by Ptolemy, had been in force for over 16 centuries with only minor changes.

Medieval cosmology differed from modern cosmology not only in its beliefs but also in the character of its explanations. The goal of theory was

[1]The present discussion was compiled from a variety of sources: Barker (1991, 1993); Baumgardt (1952); Butterfield, 1957; Gingerich (1993); Hanson (1958); Holton (1973); Koestler (1963); Koyré (1973); Kuhn (1957); Layzer (1984); Mason (1962); Stephenson (1994); Toulmin & Goodfield (1961); and Vickers (1984). Some of this material also appears in Gentner, Brem, Ferguson, Wolff, Levidow, Markman, and Forbus (1997) in a discussion of Kepler's conceptual change.

not to provide causal mechanisms but to reveal mathematical regularity and predictability. Here Kepler's path diverged from that of his predecessors. As Toulmin and Goodfied (1961, p. 198) put it, "The lifelong, self-appointed mission of Johann Kepler . . . was to reveal the new, inner coherence of the Sun-centered planetary system. His central aim was to produce a 'celestial physics,' a system of astronomy of a new kind, in which the forces responsible for the phenomena were brought to light." Holton (1973, p. 71) notes, "Kepler's genius lies in his early search for a physics of the solar system. He is the first to look for a universal physical law based on terrestrial mechanics to comprehend the whole universe in its quantitative details."[2] Kepler laid out his agenda as follows:

> I am much occupied with the investigation of the physical causes. My aim in this is to show that the celestial machine is to be likened not to a divine organism but rather to a clockwork . . . , insofar as nearly all the manifold movements are carried out by means of a single, quite simple magnetic force, . . . Moreover, I show how this physical conception is to be presented through calculation and geometry. (Kepler, in a 1605 letter to von Hohenburg, cited in Holton, 1973, p. 72)

To understand the magnitude of the conceptual change involved, an account of the prior state of belief is necessary. Western cosmology in the 16th century, continuing the tradition laid down by Plato and Aristotle and culminating in Ptolemy's system of the 2nd century AD, was roughly as follows:

1. The earth is at the center of the universe and is itself unmoving.
2. The earth is surrounded by physically real crystalline spheres, containing the heavenly bodies, which revolve around the Earth.
3. The Heavenly bodies move in perfect circles at uniform velocity.

[2]There were others, including Gilbert, in the set of early searchers, but Kepler was the first who sought to apply terrestrial physics to the universe.

(Epicycles and eccentrically positioned circles were admitted into the system to account for the observed motions.)

4. Celestial phenomena must be explained in different terms from earthly phenomena. Heavenly bodies and their spheres are made of different matter altogether. They are composed not of the four terrestrial elements—Earth, air, fire and water—but instead of a fifth element (the *quintessence)*, crystalline aether (pure, unalterable, transparent, and weightless). The farther from Earth, the purer the sphere.
5. All motion requires a mover. The outermost sphere, containing the fixed stars, is moved by an "unmoved mover," the *primum mobile.* Each sphere imparts motion to the next one in; in the Aristotelian universe, there is no empty space.
6. Celestial bodies have souls. In particular, each planet is controlled by its own spirit, which mediates its motion. (The heavenly bodies were known not to move in synchrony.)

This Aristotelian–Ptolemaic system was integrated with Catholic theology in the early 13th century by Albertus Magnus (1206–1280) and Thomas Aquinas (1225–1274). Angelic spirits were assigned to the celestial spheres in order of rank. The outermost sphere, that of the *primum mobile,* belonged to the Seraphim; next inward, the Cherubim controlled the sphere of the fixed stars; then came Thrones, Dominations, Virtues, Powers, Principalities, Archangels, and finally Angels, who controlled the sphere of the moon. The resulting conceptual scheme, dominant until the 16th century, was one of extreme intricacy, and cohesion.

Thirteen centuries after Ptolemy's model, Nicolaus Copernicus (1473–1543) published (in 1543, the year of his death) *De Revolutionibus Orbium Celestium,* proposing the idea that the Earth and other planets moved rather than the sun.[3] Copernicus argued for his system on the grounds of mathematical elegance and sufficiency, noting that the Ptolemaic system, with its vast numbers of eccentrics and epicyles, had departed

[3]Copernicus's theory was only partly heliocentric. For mathematical reasons, he placed the center of the solar system at the center of the Earth's orbit, rather than at the sun itself.

in spirit from the ancient principle of perfect circularity and regularity of movement.[4] However, Copernicus's system was not widely accepted. Even among the learned who saw the problems with the Ptolemaic system, the geocentric intuition was too strong to set aside. A more popular proposal was Tycho Brahe's system in which the five planets revolved around the sun, with the sun itself and its satellites revolving around a stationary Earth.

Kepler began as Lecturer in Mathematics at Graz in 1591, at the age of 20. He was already a confirmed Copernican, having studied the Copernican system at Tubingen with Maestlin. In his first book, *Mysterium Cosmographicum*, in 1596, he defended the Copernican view and presented his own heliocentric proposal. *Mysterium Cosmographicum* attracted the interest of Tycho Brahe (1546–1601), and in 1600 Kepler became an assistant in Tycho's observatory. When Tycho died in 1601, Kepler was appointed his successor as Imperial Mathematician of the court in Prague.

Kepler had acquired from Tycho the largest and most accurate store of astronomical observations available. He had also acquired the task of determining the orbit of Mars, a task that proved far more difficult and ultimately more revealing than Kepler had foreseen. Kepler spent the next several years trying to construct a consistent heliocentric model of the solar system based on an early version of his *equal area in equal times* assumption and on the virtually universal, self-evident principle that the orbits of the planets were (or were composed of) perfect circles. However, the fact that his calculations for Mars's orbit differed from Tycho's observations (by a mere 8° of arc) forced him to reject years of hard work and, ultimately, the ancient assumption of circularity. It is hard today to grasp how difficult it was to cast off the idea of circular orbits. Kepler in the preface to *Astronomia Nova* (Kepler, 1609/1992) commented on the "incredible" labor required to establish the existence of the solar force, largely due to his mistaken assumption of circular motion: "Because I had bound

[4]In fact, although Copernicus was able to divest his theory of the "major epicycles" that accounted for the planets' apparent retrograde motions, and of the notion of the *equant* (an imaginary point from which the calculated orbit would appear more uniform), he was forced to maintain a complex set of eccentrics and minor epicycles (Mason, 1962).

them [the planets, "the movers"] to the millstones (as it were) of circularity, under the spell of common opinion. Restrained by such fetters, the movers could not do their work" (p. 67).[5] For a seeker of mathematical regularity like Kepler, to despoil the harmony of the spheres by abandoning circular motion was a hard course to take. Far from experiencing an iconoclastic glee in overturning past beliefs, he seems to have been utterly downcast: "I have cleared the Augean stables of astronomy of cycles and spirals, and left behind me only a single cartful of dung"(cited in Koestler, 1963, p.129). After trying fruitlessly to model the planetary path with an ovoid, he at last accepted the ellipse as the shape of the orbit.[6] This led to a more precise statement of the second law of planetary motion, that a line between the sun and any planet sweeps out equal areas in equal intervals of time, and to the first law, that the planetary orbits are ellipses with the sun at one focus.[7] With this new model Kepler could replace Copernicus's (at least) 34 circles with just six ellipses.[8]

Kepler published this new view in 1609 as the *Astronomia Nova: A New Astronomy Based on Causation, or Celestial Physics.* It records his discoveries and his quest to derive the orbit of the planets—in particular, Mars, the most resistant to calculation—from causal principles.[9] This causal explanation based on mechanical forces moved him out of the kind of astronomy practiced at the time and essentially into astronomical physics. "Ye physicists, prick your ears, for now we are going to invade your

[5]Galileo (1564–1642), Kepler's brilliant contemporary and a fellow Copernican, never abandoned the belief that the planets moved in perfect circles at uniform velocity, despite receiving Kepler's evidence for elliptical orbits.

[6]After abandoning the circle, Kepler at first used the ellipse merely as a mathematical approximation to the ovoid, or egg, which had the advantage of possessing only one focus. He resisted the ellipse as a solution for physical reasons: If the sun was the unique cause of planetary motion, then there should be one unique place for it, not an arbitrary selection from between two foci as with an ellipse (Hanson, 1958, pp. 78–83).

[7]The Second Law appears in rough form in the *Mysterium Cosmographicum* (1596) and appears explicitly in Book III of the *Astronomia nova*, before the First Law in Book IV. It was in fact crucial to his derivation of the First and Third laws. The Third Law appears in the *Harmonice Mundi* in 1619.

[8]However, Kepler's system was not accepted by his contemporaries. Even those few who were willing to consider Kepler's and Copernicus's heliocentric views (including Kepler's old mentor, Maestlin) rejected his notion of a celestial physics governed by the same causal law as earthly phenomena.

[9]Hanson (1958), echoing Peirce, called Kepler's discovery of the orbit of Mars "the greatest piece of retroductive reasoning ever performed" (p. 85).

territory"(cited in Koestler, 1969, p. 325). Kepler's causal explanation of planetary motion and his three laws were a major step toward the modern conception of the solar system. According to Gingerich (1993),

> Kepler's most consequential achievement was the mechanizing and perfecting of the world system. By the *mechanization* of the solar system, I mean his insistence on "a new astronomy based on causes, or the celestial physics," as he tells us in the title of his great book. By the *perfection* of the planetary system, I mean the fantastic improvement of nearly two orders of magnitude in the prediction of planetary positions. (p. 333)[10]

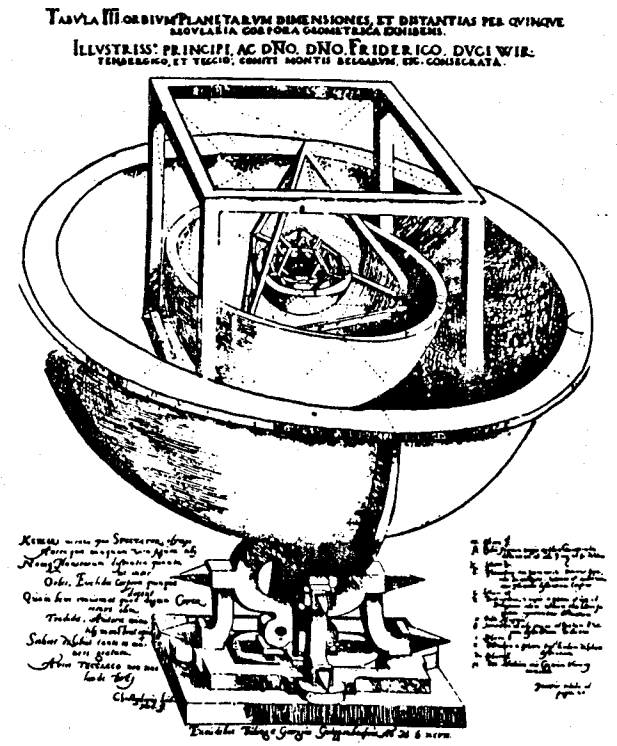
How did Kepler arrive at these discoveries? We now return to the beginning, to the *Mysterium Cosmographicum* (1596), to trace the process. Kepler had Copernicus's treatise to build on. In addition, two astronomical events had helped to prepare the ground for new conceptions of the heavens. The first was a nova (or supernova) in 1572 . This new fixed star was evidence against the Aristotelian doctrine of the unchanging and incorruptible firmament. The second was a comet in 1577 (and others not long after), whose path ran through the planetary spheres. Kepler seems to have considered this fairly conclusive evidence against the view that each planet was attached to its own crystalline sphere. He continued to ponder an alternative model, that of the Stoics, who held that the heavenly bodies were intelligent and capable of self-direction (see Barker, 1991). Throughout Kepler's writings he debated whether planetary motion required an explanation in terms of intelligent mindfulness or whether it could be ascribed to a purely physical force.

THE SUN AS PRIME MOVER: THE LIGHT/ANIMA MOTRIX ANALOGY

Kepler possessed a neo-Platonist's love of mathematical regularity, but he combined it with a commitment to explanation in terms of physical causation and an equally strong commitment to empirical test. In the preface to *Mysterium*, the 25-year-old Kepler stated his purpose as thus: "There were three things of which I persistently sought the reasons why they were such and not otherwise: the number, size and motion of the circles" (Kepler, 1596/1981, p. 63).

[10]Gingerich notes that it was the success of these predictions (the Rudophine Tables) that kept Kepler's theory alive during the two centuries after its publication.

Kepler's solution to the first two questions was a system of inscribed solids that predicted the distances of the planets from the sun. This, rather quixotic model, shown in Figure 1 clearly shows Kepler's passion for mathematic regularities. The extreme particularity of this initial model is strik-



Johannes Kepler, *Mysterium cosmographicum*. Copper engraving from the first edition (Tübingen, 1596).

Figure 1

Kepler's model of the solar system from *Mysterium Cosmographicum*, showing inscribed solids. From *Mysterium Cosmographicum*, by J. Kepler (A. M. Duncan, Trans., 1981), New York: Abaris Books. Copyright 1981 by Abaris. Reprinted with permission.

ing: The distance of a given planet from the sun could only be calculated by knowing the orbit of the next innermost planet.

The work is interesting in at least two more respects. The first is Kepler's reworking of the Copernican theory to be more consistently heliocentric. Rejecting the Copernican placement of the center of the solar system as at the center of the Earth's orbit, Kepler proposed a mathematically small but physically significant change: that the center of the solar system was the sun itself. As Aiton (1976) pointed out, Kepler's causal interpretation of Copernicus's theory led to a reaxiomitization of astronomy. Kepler also posed an important question. He noticed that the periods of the outer planets were longer, relative to those of the inner planets, than could be predicted simply from the greater distances they had to travel. That is, the planets farther away from the sun moved slower than those closer to the sun. Were the "moving souls" simply weaker in the faraway planets? Kepler reasoned thus:

> One of two conclusions must be reached: either the moving souls [*motricis animae*] are weaker the further[sic] they are from the Sun; or, there is a single moving soul [*motricem animam*[11]] in the center of all the spheres, that is, in the Sun, and it impels each body more strongly in proportion to how near it is. (Kepler, 1596/1981, p. 199)

Kepler went on to apply this hypothesis to the paths of the individual planets. If motion is caused by a single *anima motrix* in the sun that weakens with distance, this would explain why each individual planet should move slower when farther from the sun. (The arguments for this claim, of course, required recasting the observational pattern from the Ptolemaic pattern into a heliocentric system.) To reason further, he used an analogy with light (see Figure 2):

> Let us suppose, then, as is highly probable, that motion is dispensed by the Sun in the same proportion as light. Now the ratio in which light spreading out from a center is weakened is stated by the opti-

[11]Kepler's annotation in 1621 states, "If for the word "soul" [*Anima*] you substitute the word "force" [*Vim*], you have the very same principle on which the Celestial Physics is established" (*Mysterium Cosmographicum*, p. 201).

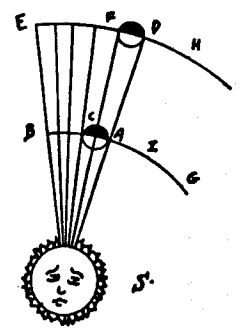


Figure 2

Kepler's depiction of the sun's light radiating outward.

> cians. For the amount of light in a small circle is the same as the amount of light or of the solar rays in the great one. Hence, as it is more concentrated in the small circle, and more thinly spread in the great one, the measure of this thinning out must be sought in the actual ratio of the circles, both for light and for the moving power [*motrice virtute*]. (Kepler, 1596/1981, p. 201)

Pushing the Analogy

Kepler returned repeatedly to the analogy between light and the motive power. In *Mysterium* (1596), the analogy functioned as a kind of existence proof that the effects of a central source could be assumed to weaken in an orderly way with distance. Kepler's many subsequent uses of this analogy served to extend and refine his notion of the anima motrix. He devoted multiple chapters of his greatest work, *Astronomia Nova* (1609), to its explanation and returned to it again in *Epitome of Copernican Astronomy* (1620). Kepler also delved further into the base domain of this analogy: the behavior of light. He published a treatise on astronomical optics (*Astronomiae Pars Optica*, 1604) and another in 1610 (*Dioptrice*). With this considerable knowledge of the behavior of light, Kepler had a base domain that was systematic and well understood and therefore ideally suited to provide inferential resources for the target (Bassok & Holyoak, 1989;

Bowdle & Gentner, 1996; Clement & Gentner, 1988; Gentner & Bowdle, 1994; Gentner & Gentner, 1983).

In *Astronomia Nova* Kepler developed this analogy much further. Early on, he challenged his motive power with the thorny question of action at a distance:

> For it was said above that this motive power is extended throughout the space of the world, in some places more concentrated and in others more spread out . . . This implies that it is poured out throughout the whole world, and yet does not exist anywhere but where there is something movable. (Kepler1609/1992, p. 382)

He answers this challenge by invoking the light analogy.

> But lest I appear to philosophize with excessive insolence, I shall propose to the reader the clearly authentic example of light, since it also makes its nest in the sun, thence to break forth into the whole world as a companion to this motive power. Who, I ask, will say that light is something material? Nevertheless, it carries out its operations with respect to place, suffers alteration, is reflected and refracted, and assumes quantities so as to be dense or rare, and to be capable of being taken as a surface wherever it falls upon something illuminable. Now just as it is said in optics, that light does not exist in the intermediate space between the source and the illuminable, this is equally true of the motive power. (Kepler, 1609/1992, p. 383)

Kepler also uses the light analogy to establish a conservation argument that the *vis motrix* is diminished with distance not through being lost but through being spread out (see Figure 3). Note his use of two further potential analogues here (odors and heat), which differ with respect to the key conservation point and serve to sharpen the parallel between light and the vis motrix.

> Since there is just as much power in a larger and more distant circle as there is in a smaller and closer one, nothing of this power is lost in traveling from its source, nothing is scattered between the source and the movable body. The emission, then, in the same man-

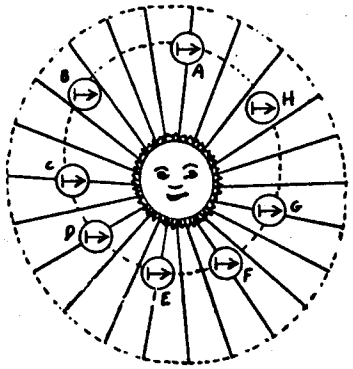


Figure 3

Kepler's depiction of the sun's motive power spreading.

> ner as light, is immaterial, unlike odours, which are accompanied by a diminution of substance, and unlike heat from a hot furnace, or anything similar which fills the intervening space. (Kepler 1609/1992, p. 381)

Modeling Analogy

To trace Kepler's analogy processes we use structure-mapping theory (Gentner, 1983, 1989) and its computational counterpart the structure-mapping engine (SME; Falkenhainer, Forbus, & Gentner, 1989). However, many of the same assumptions are shared by related models (e.g., Halford, 1993; Holyoak & Thagard, 1989; Keane, 1988). The basic idea is that analogy involves a process of alignment and projection. Assertions in a base (or source) domain are placed into correspondence with assertions in a target domain, and further assertions true of the base domain are then inferred to be potentially true of the target. Structure-mapping assumes that domain knowledge is in the form of symbolic structural descriptions that include objects, relations between objects, and higher-order relations among whole propositions. On this view, the analogical process is one of structural alignment between two mental representations to find the maximal (i.e., largest and deepest) structurally consistent match between them.

A structurally consistent match is one that satisfies the constraints of *parallel connectivity* and *one-to-one mapping* (Falkenhainer et al., 1989; Gentner, 1983, 1989; Gentner & Markman, 1993, 1994, 1997; Markman & Gentner, 1993a, 1993b, 1996; Medin, Goldstone, & Gentner, 1993). Parallel connectivity holds that if two predicates are matched, then their arguments must also match. For example, if the predicate HEAVIER(a,b) matches the predicate HEAVIER(x,y), then a must match x and b must match y. One-to-one mapping requires that each element in one representation correspond to at most one element in the other representation.

To explain why some analogies are better than others (even when factual correctness is held constant), structure-mapping uses the principle of *systematicity*—a preference for mappings that are highly interconnected and contain deep chains of higher-order relations (Forbus & Gentner, 1989; Forbus et al., 1995; Gentner, 1983, 1989; Gentner et al., 1993). Thus, the probability that an individual match will be included in the final interpretation of a comparison is higher if it is connected by higher-order relations to other systems of predicates (Bowdle & Gentner, 1996; Clement & Gentner, 1991; Gentner & Bowdle, 1994). We focus here on two predictions that derive from this framework. First, the correspondences mandated by a comparison are governed not only by local similarity but also by the degree to which the elements play the same roles in the common higher-order structure (e.g., Clement & Gentner, 1991; Gentner, 1988; Gentner & Clement, 1988; Spellman & Holyoak, 1993). Relational commonalities thus tend to outweigh object commonalities in determining the interpretation of a comparison. Second, because comparison promotes a structural alignment, differences relevant to the common structure are also highlighted by a comparison (Gentner & Markman, 1994; Markman & Gentner, 1993a, 1993b, 1996). Thus, paradoxically, comparisons can illuminate differences as well as commonalities.

SME simulates this comparison process (Falkenhainer, Forbus, & Gentner, 1986, 1989; Forbus, Ferguson, & Gentner, 1994). To capture the necessary structural distinctions we use an nth-order type predicate calculus. *Entities* stand for the objects or reified concepts in the domain (e.g., planet, orbit). *Attributes* are unary predicates used primarily to describe independent descriptive properties of objects (e.g., HEAVY(planet), which translates roughly as "The planet is heavy."). *Functions*[12] are used primarily to state dimensional properties (e.g., BRIGHTNESS(object), which translates as "the brightness of the object"). *Relations* are multiplace predicates that represent links between two or more entities, attributes, functions, or relations (e.g., REACH(anima, planet)—"The anima reaches the planet."). Relations must match identically in SME (or undergo re-representation, as discussed later), reflecting the principle that comparison is implicitly directed toward finding structural commonality. For example, REACH(light, object) could match REACH(anima, planet) but could not match DESTROY(anima, planet); that is, light reaching an object could match the anima reaching a planet, but not the anima destroying a planet. However, SME allows correspondences between nonidentical entities and dimensions (represented as functions) if they are embedded in like relational structures. Thus, the speed of a planet could be matched with the brightness of an object, provided both were governed by like relations. This is in accord with the principle that lower order information need not match identically.

This ability to match nonidentical functions is what permits cross-dimensional mappings, in everyday language ("a bright remark," "a dull book," "a lowdown scoundrel") as well as in science (see Gentner, Rattermann, Markman, & Kotovsky, 1995; Kotovsky & Gentner, in press). On the other hand, the principle of partial relational identity is equally crucial, for it captures the fact that not just any similarity constitutes an analogy. To cite an obvious example, "The sun attracts the planets" and "The nucleus attracts the electrons" are analogous, but "The sun attracts the planets" and "The sun is yellower than the moon" are not, because the second pair lacks a relational match.

It is crucial to note that the relational identity principle is not about words per se, nor does it require a total match of relational content: rather, there must be a nontrivial match of subrelations. In the analogy "The sun

[12]Functions, unlike attributes and relations, do not take truth values, but rather map objects onto other objects or values. For brevity we sometimes use the term *functor* to refer to all three categories: relations, attributes, and functions.

propels the planets as a lamp illuminates its objects;" the relations *propels* and *illuminates* are not identical, but the match is a good one nonetheless because these relations readily decompose to reveal a partial match of subrelations (over nonidentical functions, as is allowed): "The sun *increases the speed* of the planets as a light source *increases the brightness* of its recipient object." Thus, the principle of relational identity does not mean that the relations must *initially* match. Rather, it means that—through processes of abstraction or other forms of re-representation—relational identities are found as part of the analogical alignment process.

To represent beliefs about physical domains, we use qualitative process (QP) theory as a representation language (Forbus, 1984, 1990; Forbus & Gentner, 1986). (See Forbus, 1984, for a full description of the QP language and its model-building capabilities.) QP theory provides a representation language for expressing causal acccounts using qualitative mathematical relationships. For example, the statement QPROP+(*a*,*b*) expresses a positive qualitative relationship between the quantities *a* and *b*: that *a* is a monotonic positive function of (at least) *b*. QPROP-(*a*,*b*) expresses a negative qualitative relationship. The idea is to capture the psychological state of knowing the direction of change between two variables without needing to specify the exact nature of the function.

Relations can hold between expressions as well as entities. Higher-order relations between relations, such as causality, allow the construction of large representational structures: for example, the causal system for light reaching an object and the object's thereby being illuminated.

It is the presence of structurally interconnected representations that is the key to implementing structure-mapping. Given two representations in working memory, SME operates in a local-to-global manner to find one or a few structurally consistent matches. In the first stage, SME proposes matches between all identical predicates at any level (attribute, relation, higher-order relation, etc.) in the two representations. At this stage, there may be many mutually inconsistent matches. In the next stage, these local correspondences are coalesced into large mappings, called *kernels*, by enforcing structural consistency (one-to-one mapping and parallel connectivity).

SME then gathers these structurally consistent clusters into one or two global interpretations. At this point, it projects candidate inferences into the

target. It does this by projecting into the target representation any predicates that currently belong to the common structure in the base but that are not yet present in the target. These predicates function as possible new inferences imported from the base representation to the target representation. These inferences may contain new entities (skolems) that correspond to entities that initially existed only in the base. Because candidate inferences depend solely on the structure of the match, other processes are needed to evaluate their validity (see later discussion). The mappings are given a structural evaluation, reflecting the size and depth of the matching system.

SME has many useful properties for modeling conceptual change. First, the final interpretation preserves large-scale connected structure. Second, the global interpretation does not need to be explicit at the outset. The assertions that will constitute the final point of the analogy need not be present initially in the target and need not have been extracted as a separable "goal structure" or "problem-solution structure" in the base before the comparison processes begin. SME begins blindly, using only local matches, and the final global interpretation emerges through the pull toward connectivity and systematicity in the later stages of the process. Third, SME makes spontaneous inferences from its comparison process, unlike many other models of analogy (cf. Holyoak, Novick, & Melz, 1994). Finally, this model of the analogy process allows us to delineate four specific subprocesses that can change conceptual structure: highlighting, projection, re-representation, and restructuring (Gentner & Wolff, 1996; see Figure 4).

THE FOUR ANALOGICAL PROCESSES OF CONCEPTUAL CHANGE

Highlighting

SME's first result is a matching system of predicates between the base and target. This models the psychological assumption that the process of alignment causes the matching aspects of the domains to become more salient (Elio & Anderson, 1981, 1984; Gentner & Wolff, 1996; Gick & Holyoak, 1980, 1983; Markman & Gentner, 1993a, 1993b; Medin et al., 1993; Miller, 1979; Ortony, 1979). This process of highlighting is important because hu-

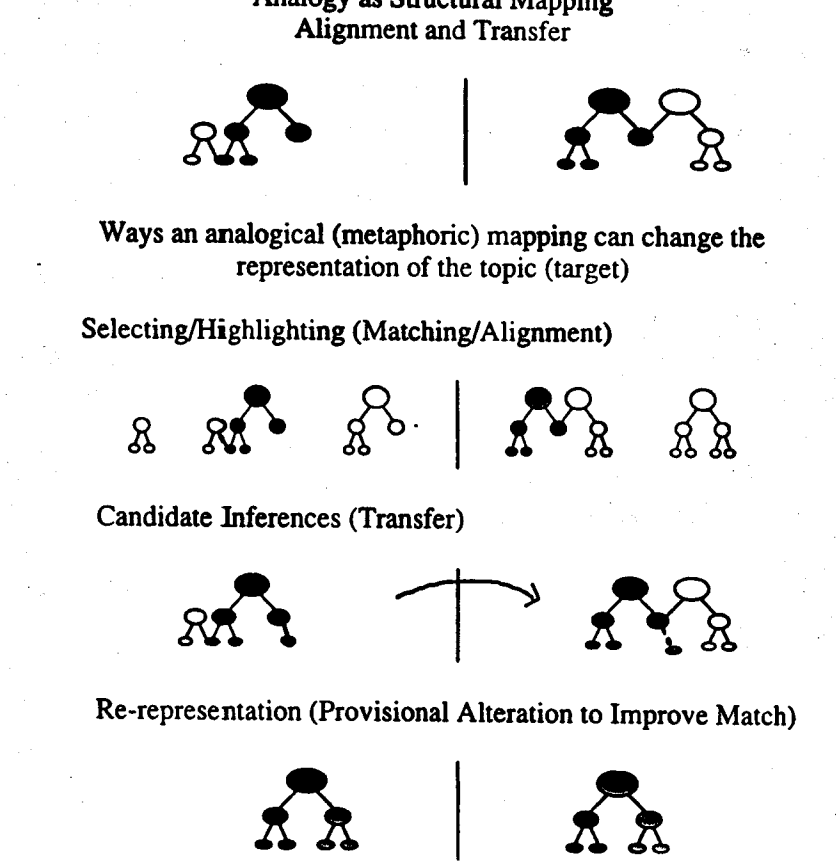


Figure 4

Ways analogy can create change.

man representations, we suggest, are typically large, rich, and thickly interwoven nets of concepts. In particular, early representations tend to be conservative, in the sense that they retain many specific details of the context of learning; that is, they are particularistic and contextually embedded (e.g., Brown, Collins, & Duguid, 1989; Forbus & Gentner, 1986; Medin & Ross, 1989). Highlighting can create a focus on a manageable subset of relevant information. Moreover, the relational identity constraint, com-

bined with re-representation processes, means that the *output* of an analogy may reveal hitherto unnoticed relational commonalities. There is considerable psychological evidence that comparison can reveal nonobvious features (Gentner & Clement, 1988; Markman & Gentner, 1993a; Medin et al., 1993; Ortony, Vondruska, Foss, & Jones, 1985; Tourangeau & Rips, 1991) and that highlighting of common information can influence category formation (Elio & Anderson, 1981, 1984; Medin & Ross, 1989; Ross, 1984, 1989; Skorstad, Gentner, & Medin, 1988).

Projection of Candidate Inferences

As described earlier, SME projects candidate inferences from the base to the target domain. These projected inferences, if accepted, add to the knowledge in the target domain. However, not all inferences made by SME will be correct. Post-mapping processes, such as the application of semantic and pragmatic constraints, are necessary to ensure the correctness of the inferences (Falkenhainer, 1990; Kass, 1994; Kolodner, 1993; Novick & Holyoak, 1991).

Re-representation

In re-representation, the representation of either or both domains is changed to improve the match. Typically, this involves a kind of tinkering in order that two initially mismatching predicates can be adjusted to match. For example, suppose an analogy matches well but for a mismatch between BRIGHTER-THAN(x,y) and FASTER-THAN(a,b) (as in Kepler's analogy between light and the vis motrix). These relations can be re-represented as GREATER-THAN(BRIGHTNESS(x), BRIGHTNESS(y)) and GREATER-THAN(SPEED(a), SPEED(b)) to allow comparison. This involves a kind of decomposition or titration that separates the GREATER-THAN magnitude relation (which is common to both) from the specific dimension of increase (which is distinctive). Studies of the development of children's comparison abilities support the psychological validity of re-representation in learning: Children are better able to match cross-dimensional analogies when they have been induced to re-represent the two

situations to permit noticing the common magnitude increase (Gentner & Rattermann, 1991; Gentner et al., 1995; Kotovsky & Gentner, in press). We return later to SME's implementation of re-representation.

Restructuring

Restructuring is the process of large-scale rearrangement of elements of the target domain to form a new coherent explanation. This rearrangement can take the form of adding or deleting causal links in the target domain as well as of altering specific concepts. It should perhaps be considered separately from the other three processes, or possibly as arising from a combination of the other three. For example, when little is known about a target domain, a mapping from the base can provide causal linkages that significantly alter the connectivity in the target. However, in the current account, there must be some minimal alignment as a basis for inference; even if no initial relational match exists, there must be at least a partial object mapping (which could be suggested by local similarities or pragmatically stipulated; Forbus & Oblinger, 1990; Holyoak & Thagard, 1989; Winston, 1980). We conjecture that substantial restructuring during a single mapping is comparatively rare, because normally the candidate inferences projected from the base domain will be at least compatible with the existing target structure. Furthermore, as Nersessian (1992, p. 24) pointed out, massive restructuring from a single base can be dangerous: She noted that Faraday's modeling of magnetic fields by analogy with the concrete lines of iron filings created by magnets led to an overly concrete, partly erroneous model of the fields. In general, we suspect that most restructuring occurs as a result of multiple analogies iteratively applied as well as other processes.

With these tools in hand, we now return to Johannes Kepler. To trace his analogical process, we represented parts of Kepler's expressed knowledge about light and the motive power. We applied SME to these representations to simulate the process of analogical reasoning that Kepler might have used in rethinking his conceptual model of the solar system.

Our representation of Kepler's knowledge of the nature of light is

shown in Figure 5.[13] Specifically, we ascribe to Kepler five beliefs: (a) A source produces light that travels instantaneously and undetectably through space until it reaches an object, at which point the light is detectable. (b) The greater the concentration of light, the brighter the object. (c) As light spreads from a source into a greater volume of space, its concentration decreases, causing the total amount of light (the product of the volume and the concentration) to remain constant. (d) Thus, the concentration of light decreases as the object's distance from the source increases. (e) Therefore, the brightness of an object decreases with distance from a source.

Kepler's initial knowledge of the motive power was of course considerably less rich than his knowledge about light. His struggle to characterize this influence is a fascinating aspect of his conceptual evolution. Early on, he called it the *anima motrix* (motive spirit), drawing on the accepted notion of intelligences governing celestial bodies. However, he was uneasy with this and strove to find a more mechanical characterization. The analogy with light, and another with magnetism (discussed later), helped him strip the sentience from the interaction between the sun and the planets. He eventually adopted the terms vis motrix or *virtus motrix* (motive force or motive power). In our representation of this knowledge (Figure 6) we use vis motrix, reflecting Kepler's shift to a less animate and more mechanical terminology.

THE VIS MOTRIX ANALOGY AND THE PROCESS OF CONCEPTUAL CHANGE

Highlighting

When given the representations of Kepler's knowledge of light and of the sun's motive force, SME produces the interpretation shown in Figure 7a. This interpretation highlights commonalities (e.g., the similarity that in both cases the emanation makes itself known when it strikes a planet and, respectively, illuminates or moves the planet).

[13]These representations are of course not intended to be exhaustive representations of Kepler's knowledge, but of the subset necessary to make our points about analogy and conceptual change. We will not attempt here a full explanation of how Kepler selected (mostly) relevant information from the larger knowledge of light. Although this is clearly important, it is beyond the scope of this paper.

5a:

```
(PRODUCE Source light)

(CAUSE (TRAVEL light Source object space)
        (REACH light object))

(INSTANTANEOUS (TRAVEL light Source object space))

(WHILE (AND (TRAVEL light Source object space)
            (NOT (REACH light object)))
       (NOT (DETECTABLE light)))

(WHILE (AND (TRAVEL light Source object space)
            (REACH light object))
       (DETECTABLE light))
```

5b:

```
(CAUSE (REACH light object)
       (PROMOTE (BRIGHTNESS object)))

(QPROP+ (BRIGHTNESS object)
        (CONCENTRATION light object))
```

5c:

```
(CAUSE (AND (QPROP+ (VOLUME light)
                    (DISTANCE Source object))
            (QPROP- (CONCENTRATION light object)
                    (DISTANCE object Source)))
       (CONSTANT (* (VOLUME light) (CONCENTRATION light object))))
```

5d:

```
(QPROP- (CONCENTRATION light object)
        (DISTANCE object Source))
```

5e:

```
(IMPLIES (AND (QPROP- (CONCENTRATION light object)
                      (DISTANCE object Source))
              (QPROP+ (BRIGHTNESS object)
                      (CONCENTRATION light object)))
         (QPROP- (BRIGHTNESS object)
                 (DISTANCE object Source)))
```

Figure 5

Representation of the light domain, the base in the light–vis motrix analogy.

6a:

```
(CAUSE (REACH vis-motrix planet)
       (PROMOTE (SPEED planet)))
```

Note: This structure matches part of the structure shown in Figure 5b for light.

6b:

```
(QPROP- (SPEED planet)
        (DISTANCE planet Sun))
```

Note: This structure matches part of the structure shown in Figure 5e for light.

Figure 6

Representation of the vis motrix, the target in the light–vis motrix analogy.

Projection

As we noted earlier, highlighting influences conceptual change in two ways: (a) by identifying relevant aspects of the two domains and thereby permitting abstraction and (b) by providing the alignable structure over which two other processes of conceptual change—*projection* and *re-representation*—can operate. This is crucial, for by constraining the candidate inferences to be those connected to the aligned structure we can model an inferential process that is generative without overshooting into "wanton inferencing."[14] The vis motrix–light analogy leads to several candidate inferences. Figure 4b shows SME's inferences, which seem reasonably like those Kepler appears to have made. First, SME infers that the vis motrix travels from the sun to the planet through space. Second, it infers that the product of volume and concentration of the vis motrix is a constant. Third, SME explains that because the concentration of the vis motrix decreases with distance and the concentration of the vis motrix governs the speed of the planet, the speed of the planet will decrease with distance from the sun. Finally, SME infers that the vis motrix will be detectable only after it reaches the planet, and not on its way. That is,

[14]Eric Dietrich (personal communication, February 1994)

7a:

```
(CAUSE (REACH light object)          (CAUSE (REACH vis-motrix planet)
   (PROMOTE (BRIGHTNESS object)))     (PROMOTE (SPEED planet)))

(QPROP- (BRIGHTNESS object)          (QPROP- (SPEED planet)
        (DISTANCE object Sun))               (DISTANCE planet Sun))
```

7b:

```
(CAUSE (TRAVEL vis-motrix Sun planet (:SKOLEM space))
       (REACH vis-motrix planet))

(CAUSE (AND (QPROP+ (VOLUME (:SKOLEM space))
                    (DISTANCE Sun planet))
            (QPROP- (CONCENTRATION vis-motrix planet)
                    (DISTANCE planet Sun)))
       (CONSTANT (* (VOLUME (:SKOLEM space))
                    (CONCENTRATION vis-motrix planet))))

(IMPLIES (AND (QPROP- (CONCENTRATION vis-motrix planet)
                      (DISTANCE planet Sun))
              (QPROP+ (SPEED planet)
                      (CONCENTRATION vis-motrix planet)))
         (QPROP- (SPEED planet) (DISTANCE planet Sun)))

(WHILE (AND (TRAVEL vis-motrix Sun planet (:SKOLEM space))
            (NOT (REACH vis-motrix planet)))
       (NOT (DETECTABLE vis-motrix)))

(WHILE (AND (TRAVEL vis-motrix Sun planet (:SKOLEM space))
            (REACH vis-motrix planet))
       (DETECTABLE vis-motrix))
```

Figure 7

Structure-mapping engine (SME) interpretation of the light–vis motrix analogy. Panel a: Interpretation (the maximal structurally consistent common structure) identified. Panel b: Candidate inferences produced for the light–vis motrix analogy. Note that the projected entity "SKOLEM space" can be filled by "space" if the vis motrix is assumed to travel through space.


```
(WHILE (AND (TRAVEL vis-motrix sun planet space)
            (NOT (REACH vis-motrix planet))
       (NOT (DETECTABLE vis-motrix))
(WHILE (AND (TRAVEL vis-motrix sun planet space)
            (REACH vis-motrix planet)
       (DETECTABLE vis-motrix)
```

Together the third and final inferences explain the phenomenon of action at a distance. These inferences can be seen in Figure 7b.

Re-representation

Earlier we suggested that the process of alignment can lead to reconstruing parts of one or both representations in such a way as to improve the alignment. Such a process may have operated on a large scale to contribute to Kepler's gradual shift toward thinking of the motive power as a physical phenomenon rather than an animistic one. However, a more locally contained example can be found shortly after the passage quoted above in the *Astronomia Nova*. Kepler here notes a discrepancy—an important alignable difference—and tries to resolve it.

> Moreover, although light itself does indeed flow forth in no time, while this power creates motion in time, nonetheless the way in which both do so is the same, if you consider them correctly. Light manifests those things which are proper to it instantaneously, but requires time to effect those which are associated with matter. It illuminates a surface in a moment, because here matter need not undergo any alteration, for all illumination takes place according to surfaces, or at least as if a property of surfaces and not as a property of corporeality as such. On the other hand, light bleaches colours in time, since here it acts upon matter *qua* matter, making it hot and expelling the contrary cold which is embedded in the body's matter and is not on its surface. In precisely the same manner, this moving power perpetually and without any interval of time is present from the sun wherever there is a suitable movable body, for it receives nothing from the movable body to cause it to be there.

> On the other hand, it causes motion in time, since the movable body is material. (Kepler 1609/1992, p. 383).

Kepler believed (according to the conventional wisdom of the time) that light moved instantaneously from the sun to light up the planets:

INSTANTANEOUS (AFFECT (light, sun, planet, space))

However, he believed that the vis motrix required time to affect the motion of the planets. At a rough level, then, Kepler faced a mismatch between the candidate inference from light (a) and his existing knowledge (b) about the planetary motion:

(a)INSTANTANEOUS (AFFECT (vis-motrix, sun, planet, space))
(b)TIME-OCCURRING (AFFECT (vis-motrix, sun, planet, space))

Kepler admits the problem but suggests a re-representation: "Although light itself does indeed flow forth in no time, while this power creates motion in time, nonetheless the way in which both do so is the same, if you consider them correctly" (Kepler, 1609/1992, p. 383). His solution is to be more precise about the notion of AFFECT (influence, planet). For such an effect to occur, he reasoned, influence must travel to the planet and influence must interact with the planet somehow. Kepler suggested that travel is instantaneous for both kinds of influences (the vis motrix and light). However, whereas light need only interact with the surfaces of bodies to illuminate them (which, according to Kepler, can be done instantaneously), the vis motrix must interact with the body of the planet itself in order to cause motion, and this requires time. Thus, Kepler gains a partial identity by decomposing and re-representing a previously problematic correspondence. Now the first part of the candidate inference can be accepted, and only the second part must be rejected.

INSTANTANEOUS (TRAVEL (light, sun, planet, space))
INSTANTANEOUS (PROMOTE (BRIGHTNESS (planet))

INSTANTANEOUS (TRAVEL (vis-motrix, sun, planet, space))
TIME-OCCURRING (PROMOTE (SPEED (planet))

Alignable Differences

Given a structural alignment, connected differences become salient. Kepler (1609/1992) used these differences to deal with the question of whether the sun's light and the motive power might not in fact be the same thing (a reasonable question, given the force of the analogy). He answered that they cannot be the same, because light can be impeded by an opaque blocker (e.g., during an eclipse), yet the motive power is not thereby impeded (otherwise motion would stop during an eclipse).

> The analogy between light and motive power is not to be disturbed by rashly confusing their properties. Light is impeded by the opaque, but is not impeded by a body . . . Power acts upon the body without respect to its opacity. Therefore, since it is not correlated with the opaque, it is likewise not impeded by the opaque . . . On this account I would nearly separate light from moving power. (pp. 392–393)

A more important alignable difference concerns the degree of decrease with distance. By the time of *Astronomia Nova*, Kepler was clear about the fact that the concentration of light diminishes as the *inverse square* of distance from its source. He therefore held himself responsible for either mapping this fact into the target, or explaining why it should not be mapped. As it happens, he still required a simple inverse law for the vis motrix, because in his model the vis motrix directly caused the planetary motion.[15]

[15]Kepler's dynamics was Aristotelian: He believed that velocity was caused by (and proportional to) the motive force, (as opposed to the Newtonian view that forces cause *changes* in velocity). He held the belief of his time, that the planets would cease to move if not pushed around the sun. Thus, he conceived the motive force as acting directly to impart counter-clockwise speed to the planets (rather than imparting inward acceleration, as in Newton's system). As Koestler (1963, p. 326) noted, Kepler had made the insightful move of decomposing planetary motion into separate components, but had reversed the roles of gravity and planetary inertia. Kepler thought that the planets' *forward* motion was caused by the sun and that their *inward–outward* motion was caused by magnetism specific to each planet. In the Newtonian system, the planets' *inward* motion is caused by the sun, and their *forward* motion is caused by their own inertia.

As usual, he tackled this discrepancy head on and produced, in *Astronomia Nova*, a long mathematical argument that, because the vis motrix can cause motion only in planes perpendicular to the sun's axis of rotation, the proper analogue to the vis motrix is light spreading out, not in a sphere around the sun, but only in a plane. Thus, he justified the alignable difference that the concentration of vis motrix should decrease as a simple inverse of distance, even though the concentration of light decreases with inverse-square distance.

Restructuring

From what we have said so far, it appears that the vis motrix analogy may have contributed to Kepler's restructuring of his model of the solar system. It provided him with a structure from which to argue for a single causal "soul" in the sun, rather than moving souls in each of the planets, and it contributed to the gradual mechanization of this soul to a power or force. The analogy may also have contributed to firming the shift from crystalline spheres containing the planets to paths continually negotiated between the sun and the planets. We return to this issue in the Discussion section.

COMPLETING THE CAUSAL ACCOUNT: THE FERRYMAN AND MAGNETISM ANALOGIES

The light–vis motrix analogy provided Kepler with the crucial inference of action at a distance. By assuming that the sun rotated around its axis (a hypothesis he confirmed by noting that sunspots move) he could account for the planetary revolutions: The planets were pushed along by a kind of circular river of force whirling around the sun, weakening with distance. However, this model still was not complete, for it did not explain how a constant force from the sun could account for the librations in the planetary orbits—that is, for the fact that the planets move inward and outward from the sun in the course of a revolution. Kepler sought a mechanism whereby the planets could somehow interact with a constant push from the sun in such a way as to capture this variation. One example was

a ferryman steering his ship in a constant current. Here the ship corresponds to the planet and the sun provides the circular river pushing the ship around (see Figure 8).[16]

> Particularly happy and better accommodated to our inquiry are the phenomena exhibited by the propulsion of boats. Imagine a cable or rope hanging high up across a river, suspended from both banks, and a pulley running along the rope, holding, by another rope, a skiff floating in the river. If the ferryman in the skiff, otherwise at rest, fastens his rudder or oar in the right manner, the skiff, carried crosswise by the simple force of the downward-moving river, is transported from one bank to the other, as the pulley runs along the cable above. On broader rivers they make the skiffs go in circles, send them hither and thither, and play a thousand tricks, without

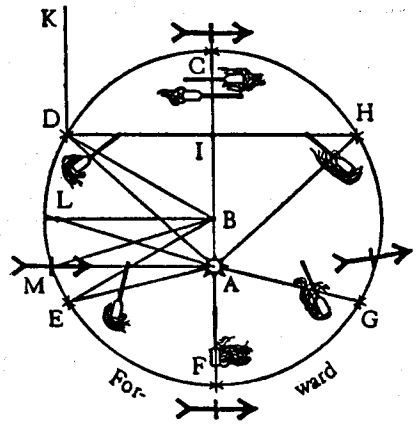


Figure 8

Kepler's analogy for the librations of the planets: The planet as ferryman in his skiff navigating in the sun's circular river; the planet as magnet interacting with the sun's magnetism.

[16]As mentioned elsewhere, in Kepler's pre-Newtonian physics the sun was required to push the planets around in their orbits, not merely to attract them.

touching the bottom or the banks, but by the use of the oar alone, directing the unified and most simple flow of the river to their own ends.

In very much the same manner, the power moving out into the world through the *species* is a kind of rapid torrent, which sweeps along all the planets, as well as, perhaps, the entire aethereal air, from west to east. It is not itself suited to attracting bodies to the sun or driving them further [sic] from it, which would be an infinitely troublesome task. It is therefore necessary that the planets themselves, rather like the skiff, have their own motive powers, as if they had riders [*vectores*] or ferrymen, by whose forethought they accomplish not only the approach to the sun and recession from the sun, but also (and this should be called the second argument) the declinations of latitudes; and as if from one bank to the other, travel across this river (which itself only follows the course of the ecliptic) from north to south and back. (Kepler, 1609/1992, p. 405)

The Magnetism Analogy

Although Kepler returned to the ferryman analogy from time to time, this analogy was unsatisfying, in part perhaps because it seemed to require too much insight from the planets. How would they know when to shift the rudder? In keeping with a lifelong quest to explain seemingly intelligent planetary behavior in terms of a mechanical interaction, Kepler sought to explain the planet's behavior purely physically. He wanted the ship without the ferryman.

Kepler's longest and most determined effort in this direction was the use of an analogy between the vis motrix and magnetism, an analogy Kepler developed over a long period. Kepler had first mentioned the magnetism analogy in the *Mysterium* (1596) as one more instance of action at a distance that might make his sun–planet force more plausible. By the time of the *Astronomia Nova* (1609) Kepler had become familiar with the work of William Gilbert (*De Magnete*, 1600/1938). In addition to setting forth the properties and behaviors of magnets, Gilbert had conjectured that the Earth might function as a giant magnet. Kepler extended this analogy to

the sun and planets. Not only was magnetism another example of action at a distance, it also had the potential to explain the variations in distance. By modeling the planets and the sun as magnets, Kepler thought he could explain the inward and outward movements of the planets in terms of attractions and repulsions resulting from which poles were proximate.

In the *Epitome of Copernican Astronomy* (1621/1969), Kepler presented a long discussion of magnetism and its analogy to the planetary system. He began with a simple version of the magnetism analogy, likening the Earth to iron filings and the sun to a lodestone (magnet). This analogy, mentioned only briefly, establishes a second example of action at a distance, in that a lodestone affects the behavior of iron filings without ever making contact with the filings. In addition, like the light–vis motrix analogy, it suggests that action at a distance produces a qualitatively negative relationship between the influence of one object over another and distance. Gilbert had established this relationship between distance and magnetic influence in *De Magnete*. However, it does not explain why the planets would move closer and farther away from the sun, as iron filings would be uniformly attracted to a lodestone. Indeed, according to the iron filings analogy, the planets should be dragged into the sun.

In the second analogy, Kepler conceived of the planet as a magnet (or lodestone). This adds some new inferential power to the magnetism analogy. Kepler could now use the attractive and repulsive forces between the different poles of a pair of magnets to explain the coming together and separating of the celestial bodies. Thus, the planet would move closer to the sun when its attractive pole was turned toward the sun, and farther from the sun when the repelling pole was turned toward the sun (see Figure 9a). Given this varying distance from the sun, the planet's varying speed could also be inferred (as Kepler had already established that the planets move faster when closer to the sun—by the light–vis motrix analogy, and his second law).

Kepler was unsure whether the lodestone–vis motrix correspondences were merely analogical or actually represented an identity. He struggled with this issue throughout the *Astronomia Nova*. Early in the treatise, he wrote the following:

> The example of the magnet I have hit upon is a very pretty one, and entirely suited to the subject; indeed, it is little short of being the very truth. So why should I speak of the magnet as if it were an example? For, by the demonstration of the Englishman William Gilbert, the earth itself is a big magnet, and it is said by the same author, a defender of Copernicus,[17] to rotate once a day, just as I conjecture about the sun. And because of that rotation, and because it has magnetic fibres intersecting the line of its motion at right angles, those fibres lie in various circles about the poles of the earth parallel to its motion. I am therefore absolutely within my rights to state that the moon is carried along by the rotation of the earth and the motion of its magnetic power, only thirty times slower. (Kepler 1609/1992, chap. 34, pp. 390–391)

Later, he voiced the concern that there are significant differences between the vis motrix and magnetism, and that they therefore cannot be equated:

> I will be satisfied if this magnetic example demonstrates the general possibility of the proposed mechanism. Concerning its details, however, I have my doubts. For when the earth is in question, it is certain that its axis, whose constant and parallel direction brings about the year's seasons at the cardinal points, is not well suited to bringing about this reciprocation or this aphelion . . . And if this axis is unsuitable, it seems that there is none suitable in the earth's entire body, since there is no part of it which rests in one position while the whole body of the globe revolves in a ceaseless daily whirl about that axis. (Kepler, 1609/1992, p. 560)

Yet despite these concerns, Kepler continued to use the phrase "magnetic force" or "magnetic *species*" to describe the vis motrix throughout the text. One reason that he did so may be that the only alternative he could think of to a magnetic force was a mind in the planet, one that would somehow perceive the planet's distance from the sun (perhaps by registering the sun's

[17]Kepler was incorrect here. Although Gilbert believed that the Earth rotated on its axis, he retained a Tychonic model in which the sun and its satellite planets revolve around the Earth.

apparent diameter) and move accordingly. Kepler's desire to reduce or replace this intelligence with a mechanical force is a recurring theme in his analogies.

ADDITIONAL ASPECTS OF CREATIVE ANALOGY

We have mentioned highlighting, candidate inferences, re-representation, and restructuring as mechanisms of analogical learning. In addition, we believe at least three additional mechanisms are needed to capture creative analogy processing. First, a mechanism is needed to mediate between multiple analogies such as the magnetism analogy, the light analogy, and the ship analogy. One computational approach might be found in Burstein's (1986) CARL, which combined different analogies to build a representation of how a variable works. Spiro, Feltovich, Coulson, and Anderson (1989) have also traced the way in which multiple analogies interact (not always peacefully) in learning complex domains.

A second mechanism needed is incremental analogizing. As new information about a domain is learned or brought in, the learner must be able to extend the original mapping. It has been shown that individuals are sensitive to a recent mapping and will more quickly extend that mapping than create a new one (Boronat & Gentner, 1996; Gentner & Boronat, 1992; Gibbs & O'Brien, 1990; Keane, 1990). Keane and Brayshaw's (1988) simulation was the first to capture the finding that people's initial mappings influence the subsequent correspondences they can readily draw. We have adapted Keane and Brayshaw's technique to create an incremental version of SME, called Incremental Structure-mapping Engine (ISME), which can extend an analogy after the initial mapping. It draws further information from its long-term knowledge about the base and target to add to the working memory descriptions. It then re-maps the analogy, building on the results of the initial mapping and thus enriching the overall analogical mapping (Forbus et al., 1994). ISME can model the process of extended analogizing in problem solving. Could it partially explain creative extension processes like Kepler's? We address this question in the Discussion section.

A third mechanism—or combination of mechanisms—is one that can test the projected inferences of the mapping and make re-representations when needed. The notion of re-representation in analogical reasoning has recently been a focus of attention in analogy and case-based reasoning research (Kass, 1994; Keane, 1988; Kolodner, 1993; Novick & Holyoak, 1991). Falkenhainer's (1990) PHINEAS system has an adaptation step as part of an analogical discovery process. It constructs physical theories by analogy with previously understood examples, by iterating through what Falkenhainer called a *map/analyze cycle.* In this cycle, PHINEAS starts with a qualitative description of a physical system's behavior and a set of domain theories. If it does not have an applicable theory to explain the new behavior, it uses analogy to find an explanation. PHINEAS has an index of previously explained examples, arranged using an abstraction hierarchy of observed behaviors. PHINEAS selects and evaluates potentially analogous examples from this hierarchy and then uses SME to generate a set of correspondences between the novel behavior and the understood example. The explanation for the new behavior is then projected from the explanation of the old behavior. PHINEAS then tests this new explanation to make sure that it is coherent with its rules about physical domains. When there is conflict, PHINEAS can re-represent some predicates. It then simulates the operation of the new theory to see if the newly mapped structure in the target can produce the observed behavior.

DISCUSSION

Kepler used analogies both widely and deeply in his quest for an understanding of planetary motion. We have traced some of these analogies and modeled the processes using SME. We suggest that these analogies were instrumental to Kepler's conceptual change. Let us begin by justifying some key assumptions.

Did Kepler Use Analogy in Thinking?

The frequent use of analogies in Kepler's texts is no guarantee that these analogies drove his conceptual change. He could have used analogy solely

as a rhetorical device. Although there is no way to decide this issue definitively, there are reasons to believe that at least some of Kepler's analogies were instrumental in his thought processes. First, as discussed earlier, the open and inclusive character of Kepler's writing, and his apparent insistence on taking the reader through his tortuous course of discovery, suggest that the extended analogies he provided were actually used in his thought processes. Second, and more directly, Kepler's major analogies were pursued with almost fanatical intensity across and within his major works. There are numerous detailed diagrams of base and target, and long passages that spell out the commonalities, the inferences, and the incremental extensions, as well as alignable differences between base and target and Kepler's assessment of their import. Furthermore, for both his major analogies, he delved energetically into the base domain: reading Gilbert's *De Magnete* in the case of magnetism and writing his own treatise, *Astronomiae Pars Optica,* in the case of light. Kepler's long discussions about the status of the magnetism–vis motrix comparison—whether it was purely an analogy or might instead in fact be the causal means by which the sun influenced the planets—are another indication of the seriousness with which Kepler took his analogies.

A third indication that Kepler might have used analogies in thinking is the sheer fecundity of his analogizing, which suggests that analogy was a natural mode of thought for him. In pursuit of a causal model of the planetary system, Kepler analogized sun and planet to sailors in a current, magnets, a balance beam, and light, to name only some of the more prominent analogies (see Figure 9). For example, in the *Epitome* (Kepler, 1621/1969) he compared his celestial physics—in which planetary paths arise out of interacting forces—with the fixed-firmament theories of the ancients:

> Here we entrust the planet to the river, with an oblique rudder, by the help of which the planet, while floating down, may cross from one bank to the opposite. But the ancient astronomy built a solid bridge—the solid spheres—above this river,—the latitude of the zodiac—and transports the lifeless planet along the bridge as if in a chariot. But if the whole contrivance is examined carefully, it appears that this

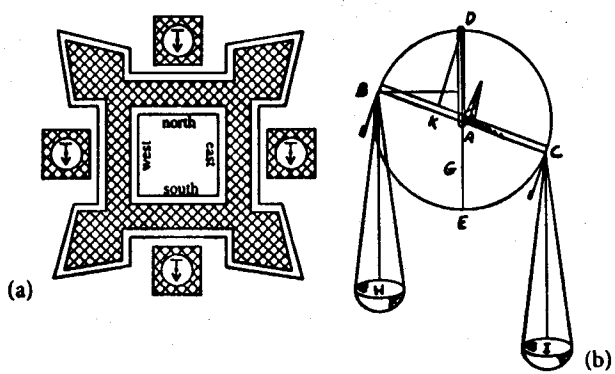


Figure 9

Examples of Kepler's multiple analogies for planetary motion: (a) magnet and (b) balance scale.

> bridge has no props by which it is supported, nor does it rest upon the earth, which they believed to be the foundation of the heavens. (pp. 182–183)

Analogies were used for matters personal as well as public. For example, Kepler complained of the astrological forecasts he often had to cast: "A mind accustomed to mathematical deduction, when confronted with the faulty foundations [of astrology] resists a long, long time, like an obstinate mule, until compelled by beating and curses to put its foot into that dirty puddle" (Kepler, 1606, in *De Stella Nova in Pede Serpentarii*, quoted in Koestler, 1963).

In another engaging passage, Kepler (1609/1992) introduced the *Astronomia Nova* to his royal patron with a long, elegant analogy treating his solution to Mars's orbit as a kind of capture of war:

> I am now at last exhibiting for the view of the public a most Noble Captive, who has been taken for a long time now through a difficult and strenuous war waged by me under the auspices of Your Majesty . . . It is he who is the most potent conqueror of human inventions, who, ridiculing all the sallies of the Astronomers, escaping their devices, and striking down the hostile throngs, kept safe

> the secret of his empire, well guarded throughout all ages past, and performed his rounds in perfect freedom with no restraints: hence, the chief complaint registered by that Priest of Nature's Mysteries and most distinguished of the Latins, C. Pliny, that "Mars is the untrackable star" . . . In this place chief praise is to be given to the diligence of Tycho Brahe, the commander-in-chief in this war, who . . . explored the habits of this enemy of ours nearly every night for twenty years, observed every aspect of the campaign, detected every stratagem, and left them fully described in books as he was dying . . . I, instructed by those books as I succeeded Brahe in this charge, first of all ceased to fear [the enemy] whom I had to some extent come to know, and then, having diligently noted the moments of time at which he was accustomed to arrive at his former positions, as if going to bed, I directed the Brahean machines thither, equipped with precise sights, as if aiming at a particular target, and besieged each position with my enquiry. (pp. 30–35)

Clearly, Kepler liked to play with analogies. But there is a fourth reason to assume that he used analogies in his thinking, namely, that he explicitly stated that he did so. For example, Vickers (1984) discussed how in the Optics (1904) Kepler treated the conic sections by analogy with light through a lens and justified this unorthodox treatment thus:

> But for us the terms in Geometry should serve the analogy (for I especially love analogies, my most faithful masters, acquainted with all the secrets of nature) and one should make great use of them in geometry, where—despite the incongruous terminology—they bring the solution of an infinity of cases lying between the extreme and the mean, and where they clearly present to our eyes the whole essence of the question. (pp. 149–150)

A more specific reference to analogy in Kepler's creative thinking occurs in his own writings about how he originally arrived at the anima motrix idea. In his 1621 annotations to *Mysterium Cosmographicum*, Kepler commented explicitly on the role of analogy in his knowledge revision

process. In the original version, in 1596, he had argued that there was "a single moving soul [*motricem anima*] in the center of all the spheres, that is, in the Sun, and it impels each body more strongly in proportion to how near it is" (Kepler, 1596/1981, p. 199). In 1621, he wrote the following:

> If for the word "soul" [*Anima*] you substitute the word "force" [*Vim*], you have the very same principle on which the Celestial Physics is established . . . For once I believed that the cause which moves the planets was precisely a soul. . . . But when I pondered that this moving cause grows weaker with distance, and that the Sun's light also grows thinner with distance from the Sun, from that I concluded, that this force is something corporeal, that is, an emanation which a body emits, but an immaterial one. (Kepler, 1621/1969, p. 201)

What Did Kepler Mean by *Analogy*?

A fifth indication that Kepler took analogy seriously as a tool for thought is that he devoted some energy to discussing its proper use in thinking. He lived in a curious time with respect to the use of analogy and metaphor in discovery. The alchemists', the dominant approach to scientific phenomena, was still a major force in medieval Europe during Kepler's life. The alchemists were remarkable, from the current point of view, both in their zeal for using metaphors and analogies to explain natural phenomena and in their manner of using them. From the viewpoint of current scientific practice, their use of analogy was unrestrained, bordering on the irrational (see Gentner & Jeziorski, 1993, for a comparison of alchemical analogizing with current scientific practice). Kepler, who rarely engaged in collegial tussling, was sharply critical of this sort of analogizing. In the *Harmonice Mundi* (1619) he attempted to distinguish the proper use of analogy from the methods of alchemists, hermeticists, and others of that ilk: "I have shown that Ptolemy luxuriates in using comparisons in a poetical or rhetorical way, since the things that he compares are not real things in the heavens" (cited in Vickers, 1984, p. 153). In a letter to a colleague in 1608, Kepler attempted to make explicit the qualities that make for useful analogizing:

> I too play with symbols, and have planned a little work, Geometric Cabala, which is about the Ideas of natural things in geometry; but I play in such a way that I do not forget that I am playing. *For nothing is proved by symbols . . . unless by sure reasons it can be demonstrated that they are not merely symbolic but are descriptions of the ways in which the two things are connected and of the causes of this connexion.* (cited in Vickers, 1984, p. 155)

Kepler believed, then, that analogy is heuristic, not deductive. His second (italicized) statement sounds remarkably like a modern cognitive view: He seems to be suggesting, as we do in this chapter, that the two domains should contain the same system of relationships and causal structures (although he might also have meant that there should be causal connections between the two domains analogized). For our purposes, the key point is that he explicitly concerned himself with the proper use of analogy in thinking.

Analogy and Creativity

One indication of creativity is the magnitude of the change in ideas. Kepler's ideas changed radically over the course of his life. Many of these changes had multiple contributors, including Bruno, Copernicus, Tycho, Gilbert, Galileo, and others. However, much of the change occurred as a result of Kepler's own creative thought processes.

1. Formerly, the paths of the planets were composed of perfect circles and the planets moved at uniform velocity. Over the course of his work, Kepler shifted to the belief that the planets move in ellipses with the sun at one focus, faster when closer to the sun and slower when farther. This was a far more radical change than most of us can today

appreciate: "Before Kepler, circular motion was to the concept of a planet as 'tangibility' is to our concept of 'physical object' " (Hanson, 1958, p. 4).

2. Formerly, the planets' orbits were conceived of either as crystalline spheres containing the planets or as eternal paths, composed of circles, traveled by planetary intelligences. Kepler came to see them as paths continually negotiated between the sun and the planets. As Toulmin and Goodfield (1961) noted, "One cannot find before Kepler any clear recognition that the heavenly motions called for an explanation in terms of a *continuously* acting physical force" (p. 201, emphasis in original).
3. Formerly, celestial phenomena were considered completely separate from earthly physics. Kepler freely extended terrestrial knowledge to astronomical phenomena. He applied analogies from the domains of light, magnetism, balance scales, sailing, and the optics of lenses, among many others.
4. Formerly, the planetary system was governed by mathematical regularities. Kepler changed it to one governed by physical causality *and* a resulting mathematical regularity. As noted by Gingerich (1993),

> Copernicus gave the world a revolutionary helio*static* system, but Kepler made it into a heliocentric system. In Kepler's universe, the Sun has a fundamental physically motivated centrality that is essentially lacking in *De revolutionibus*. We have grown so accustomed to calling this the Copernican system that we usually forget than many of its attributes could better be called the Keplerian system. (p. 333)

5. Early in Kepler's work, he proposed the anima motrix as the "spirit" in the sun that could move the planets. Later, he called it the vis motrix or virtus motrix. This change could be considered an ontological change, an instance of what Thagard (1992) calls "branch jumping." It could also be analyzed as differentiation (Smith, Carey, & Wiser, 1985), analogous to the notion of "degree of heat," which differentiated into heat and temperature (Wiser & Carey, 1983).

However, in Kepler's case the split is somewhat more dramatic: An early animate–mechanistic notion differentiated or specialized into a purely mechanical notion.[18] This change marked a shift toward a mechanization of planetary forces.

One other change is harder to sum up. Early in Kepler's work, the planets (in the Stoic account—the leading account after the crystalline spheres had been punctured by Tycho's comet)—were intelligences (Barker, 1991). Kepler struggled with the notion of a planetary intelligence throughout his career. Kepler had to find a way of thinking about the planets that could predict their individual behaviors, while assigning to them the minimal possible number of animate or entient properties. Lacking any established notion of force, Kepler had to develop these ideas by gradually stripping away from the notion of "intelligence" more and more of its normal properties. For example, he asked himself whether he could explain the fact that planets go faster when nearest the sun by granting them only the ability to perceive the sun's diameter.

How should we characterize the magnitude of these changes? Theories of knowledge change distinguish degrees of alteration in the existing structure (e.g., Carey, 1985; Thagard, 1992). *Belief revision* is a change in facts believed. *Theory change* is a change in the global knowledge structure. *Conceptual change*, in some sense the most drastic, is a change in the fundamental *concepts* that compose the belief structure. Conceptual change thus requires at least locally nonalignable or incommensurable beliefs (Carey, 1985). Of the changes just mentioned, we suggest that most if not all of them would qualify as theory change, and that Statements 2 and 5 have a good claim to be full-fledged changes of concepts.

Our results indicating that Kepler used analogies in his creative thinking accord with other work on the history of science. The journals of such great contributors to the scientific enterprise as Boyle, Carnot, Darwin,

[18]The prevalence of these combined animate–mechanistic notions in the historical record is worth considering in the context of theories of cognitive development. They suggest that distinctions between domains—and even the theory of what constitutes a domain—are culturally defined rather than innately present.

Faraday, and Maxwell contain many examples of generative uses of analogy (Darden, 1992; Gentner, 1982; Gentner & Jeziorski, 1993; Nersessian, 1985, 1986, 1992; Nersessian & Resnick, 1989; Ranney & Thagard, 1988; Thagard, 1992; Tweney, 1983; Wiser, 1986; Wiser & Carey, 1983). Modern creative scientists such as Robert Oppenheimer (1956) and Sheldon Glashow (1980) have commented explicitly on the usefulness of analogy in their work. Finally, direct field observations of molecular biologists at work (Dunbar, 1995) and case studies in the history of psychology (Gentner & Grudin, 1985; Gigerenzer, 1994) demonstrate that analogy is frequently used in the everyday practice of science.

Kepler Compared With Current Scientists

It is useful to compare Kepler's use of analogy with that observed by Kevin Dunbar (1995, this volume) in his observations of microbiology laboratories. Dunbar suggests three factors that make for a productive laboratory: frequent use of analogy, attention to inconsistency, and heterogeneity of the research group. Dunbar's working question is, of course, quite different from ours; there need be no necessary connection between what makes for a creative laboratory and what makes for a creative individual. Nonetheless there are some striking commonalities. Dunbar's detailed analyses show that the highly productive microbiology laboratories are those that use analogies in quantity and take them seriously. In the successful lab groups, analogies are extended and "pushed" in group discussions. This is the most direct evidence to date that the process of working through an analogy contributes to scientists' on-line creative thinking, and it lends force to Kepler's introspection that analogy furthered—and perhaps even engendered—his theories. Another possible parallel stems from Dunbar's observation that the heterogeneity of the laboratory group contributes to creativity. Dunbar speculates that this is true in part because group heterogeneity increases the range of different analogues that can be brought to bear. The idea that a stock of different analogues is conducive to creative thought accords with our conclusions concerning Kepler. (However, we suggest that such multiplicity is helpful only if the individual analogies are dealt with energetically.) The mode of thought in which one

slides and blends freely across different analogues is rarely as successful in scientific analogy as it is in expressive metaphor. This is because it typically undermines structural consistency and hence the inferential usefulness of an analogy (Gentner, 1982; Gentner & Jeziorski, 1993; Markman, 1996). Kepler seems to have profited considerably from working through the magnet and light analogues for the sun's motive power and from exploring parallels between them.

There are also commonalities not directly related to analogy. Attention to inconsistencies is another factor Dunbar singles out in his analysis of creative laboratories. Kepler worried about inconsistencies and was driven by them to keep pushing old analogies and in some cases to reject them. However, we suggest that these two factors play different roles. Attention to inconsistencies is a *motivator* of conceptual change, whereas analogy is a *process* by which conceptual change occurs.

There are also some interesting (alignable) differences between the patterns Dunbar observed and Kepler's recorded patterns. First, by far the majority of the analogies Dunbar observed are close literal similarities (what he calls *local analogies*), typically involving the same kind of organism or species, similar diseases or genetic materials, and so forth. Kepler did in fact use close analogues on many occasions. When he first noticed the key pattern that speed diminished as distance from the sun increased, he immediately applied this between-planet pattern within planets to suggest that each planet moves fastest when it is closest to the sun. This led to the first statement of the equal-areas law. As another example, his calculation of Mars's orbit depended on the reverse analogy of imagining how the Earth's orbit would appear from Mars. Again, he tested his reasoning about the sun and planets by applying that same reasoning to the Earth and its satellite moon, which he regarded as closely analogous to the sun and its satellite planets.

However, in contrast to the microbiologists, Kepler also used many distant analogies. We believe this stems in part from the different historical stages of the disciplines. Kepler was forming the new science of astrophysics, more or less in the absence of a usable physics. Given this underdeveloped state of affairs, distant analogies were in many cases his only

option. There was no literal similarity to be had. In contrast, in the microbiology laboratories that Dunbar studied, the historical moment is one of a fairly well agreed-on (but not yet fully explored) framework in which there are many close analogues (similar cases) that are likely to be extremely fruitful. That is, we suspect that close analogies and far analogies may be useful at different stages in the history of a field. Local analogies are useful for filling in a framework, whereas distant analogies are used for developing a new framework.

Finally, another marked difference turns on another aspect of Dunbar's third claim: that creative labs have social interaction patterns that bring together heterogeneous knowledge. Clearly, this cannot apply literally to Kepler, who worked alone. (Though his correspondence shows steady efforts to find collegial interactions, his contemporaries on the whole found his work too radical, or too mystifying, to accept.) Should we then think of Kepler as a kind of one-man equivalent of Dunbar's heterogeneous groups, who produced a large variety of analogies and therefore a good pool of possible solutions?

This brings us to the final question. Is it possible to say exactly wherein Kepler's analogizing differed from the way in which ordinary people do analogy? The short answer, of course, would be that he was a creative genius and most people are not. But let us attempt to be more specific. One obvious partition within analogical processing is that between (a) the *retrieval* processes by which potential analogues are accessed from memory and (b) the *mapping* processes that go on after both analogues are present. There is substantial evidence for a disassociation between those two (Gentner, 1989; Gentner, Rattermann, & Forbus, 1993). Similarity-based retrieval to a probe (Process a) is considerably less discerning and structure-sensitive than is comparison of two present items (Process b). Similarity-based access to long-term memory typically produces mundane literal similarity matches or even matches that are surface-similar but not structurally similar (Gentner, Rattermann, & Forbus, 1993; Gick & Holyoak, 1980; Holyoak & Koh, 1987; Keane, 1988; Reeves & Weisberg, 1994; Ross, 1989). Yet once both analogues are present, people typically show a high degree of sensitivity to structure and can fluently carry out

abstract mappings (Clement & Gentner, 1991; Gentner, Rattermann, & Forbus, 1993; Holyoak & Koh, 1987). For example, when given analogies to use in solving problems, people are typically fairly selective about choosing analogies that have genuine structural overlap with the target problem (Bassok & Holyoak, 1989; Holyoak & Koh, 1987; Novick & Holyoak, 1991; Novick & Tversky, 1987; Ross, 1987; Ross, Ryan, & Tenpenny, 1989). We have simulated subjects' retrieval patterns with the MAC/FAC simulation (Many Are Called but Few Are Chosen), in which a first stage retrieval process carries out a wide, computationally cheap and structurally insensitive search for candidate retrievals and a later stage (the FAC stage, essentially SME) performs a structural alignment over these candidate analogues (Forbus, Gentner, & Law, 1995). This system does a good job of capturing the phenomena: Retrievals based on surface similarity or on overall similarity are common, and retrievals based on purely relational similarity—the purely analogical retrievals that strike us as clever and creative—also occur, but rarely.

Thus we might be tempted to conclude that Kepler differs most radically from others in his fertile access to various prior analogues (Process a). It is certainly plausible that what most distinguishes highly creative thinkers is a high rate of spontaneous analogical retrievals. But another possibility worth considering is that it may be the mapping process (Process b) that most differentiates highly creative individuals. There are two reasons for this speculation. First, the more energetic the mapping process, the more each analogy is likely to reveal its full potential set of inferences. Second, and less obvious, we conjecture that intense mapping may promote fertile access. For if access to prior material depends on a common encoding (Forbus, Gentner, & Law, 1995), then the highlighting, inferencing, and re-representing carried out in the course of pushing analogies may benefit subsequent memory access, as such activities tend to increase the scope of common internal representations (Gentner, Rattermann, Markman, & Kotovsky, 1995). This account is consonant with Seifert, Meyer, Davidson, Patalano, & Yaniv's (1995) prepared-mind perspective on creativity, and also with Gruber's (1995) discussion of Poincaré. Thus, we suggest that a major reason for Kepler's high rate of ana-

logical remindings is the intensity of the alignment processes he carried out on his analogies once he had them in working memory.

A further contributor to analogical fecundity might be heterogeneity of interests within an individual. Kepler's published works, besides his great works on celestial physics, included papers on optics, the nature of comets, the birthdate of Jesus, and a new method of measuring wine casks (in which he developed a method of infinitesmals that took a step toward calculus). Such diversity within an individual might be analogous to the heterogeneity of background Dunbar noted in his successful laboratory groups.

Creativity and Structure

It is a common intuition that creativity is the opposite of rigidity, that it is characterized by fluid concepts and shifting relationships and unclassifiable processes. In contrast, we have suggested that at least some kinds of creativity are better described as structure-sensitive processes operating over articulated representations. For example, SME, a system that thrives on structured representations, behaves in what might be considered to be a creative manner when it notices cross-dimensional structural matches, projects candidate inferences, infers skolomized entities, and (for ISME) incrementally extends its mapping. We suggest that analogy is an engine of creativity in part *because* it provides a fair degree of structure while inviting some alteration.

Interestingly, in a creative drawing task, Ward (1994) noted similar self-generated structure-preserving strategies. He showed that when individuals are asked to create new instances of a category, their drawings tend to rely closely either on exemplars they have just seen (Smith, Ward, & Schumacher, 1993) or on self-generated category standards (Cacciari, Levorato, & Cicogna, this volume; Ward, 1994). This is true even when individuals are explicitly told to create items that are very different. Ward et al. (1995) stressed the heavy reliance on prior structures in the creative process.

Totally fixed structure can lead to rigidity, but total fluidity is equally inimical to creative change because in such a condition no state can be interestingly distinguished from any other. To close with an analogy, the difference between two quartz crystals is interesting, but the difference between two configurations of molecules in a glass of water is not. We suggest that creativity is more like crystal building than it is like fluids shifting: It is only when the structures are intricate enough to be distinguishable that we care when a change has occurred. On this analogy, creativity is best seen when there is a large enough structure to permit significant change. The example of Kepler is more consonant with the notion of creativity as structural change than it is with creativity as fluidity. There are probably many paths to creativity. Kepler's writings reveal that analogy is one of them.

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