COGNITIVE SCIENCE A Multidisciplinary Journal



Cognitive Science 34 (2010) 752–775 Copyright © 2010 Cognitive Science Society, Inc. All rights reserved. ISSN: 0364-0213 print / 1551-6709 online DOI: 10.1111/j.1551-6709.2010.01114.x

Bootstrapping the Mind: Analogical Processes and Symbol Systems

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Received 16 December 2009; received in revised form 11 May 2010; accepted 11 May 2010

Abstract

Human cognition is striking in its brilliance and its adaptability. How do we get that way? How do we move from the nearly helpless state of infants to the cognitive proficiency that characterizes adults? In this paper I argue, first, that analogical ability is the key factor in our prodigious capacity, and, second, that possession of a symbol system is crucial to the full expression of analogical ability.

Keywords: Analogical learning; Cognitive development; Structure-mapping; Language and cognition; Bootstrapping

1. Introduction

What makes us so smart as a species, and what makes children such rapid learners? I argue that the answer to both questions lies in a mutual bootstrapping system composed of (a) our exceptional capacity for relational cognition and (b) symbolic systems that augment this capacity. The ability to carry out structural alignment and processing is inherent in human cognition. It is arguably the key inherent difference between humans and other great apes (Gentner, 2003; Gentner & Christie, 2008; Penn, Holyoak, & Povinelli, 2008; Thompson & Oden, 1996). But an equally important difference is that humans possess a symbolic language (Gentner, 2003; Gentner & Christie, 2008). Language is a powerful force in cognitive development. As Carey (1985b, 2009) has emphasized, it provides "tools of wide application" that contribute to children's cognitive abilities.

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My claim is that structural alignment supports language learning, and that in turn language—especially relational language—supports structural alignment and reasoning. In sum, language forms a positive feedback system with relational cognition; and this system is the major driver of specifically human learning.

In this paper, I argue for three specific claims that bear on this larger thesis.

- 1. Comparison involves a process of structural alignment and mapping that leads to learning via abstraction, rerepresentation, inference-projection, and difference-detection.
- 2. Language interacts with structure-mapping in a mutual bootstrapping process.
- 3. The same structure-mapping process operates over close, mundane comparisons as over more distant analogical comparisons. Further, close comparisons potentiate more distant, purely relational comparisons through *progressive alignment*.

I am not saying that this system is the only way in which humans learn. We learn in many other ways too—for example, by association, by reinforcement, by automatization, and by statistical learning of sequences. But these capabilities are shared with other animals. My claim is that the positive feedback system between language and structure-mapping processes is what makes humans uniquely powerful learners.

I begin with a brief review of the structure-mapping process and how it promotes learning, with illustrative studies. Then, I describe how language—particularly relational language—interacts with analogical processing in children's learning. Finally, I apply these ideas to an analysis of Carey's account of the acquisition of the natural numbers, in which the interaction between language and analogical mapping plays a central role.

2. Structure-mapping

According to structure-mapping theory (Gentner, 1983), human comparison¹ involves a process of establishing a *structural alignment* between two represented situations² and then projecting *inferences*. The commonalities and differences between two situations are found by determining the maximal structurally consistent alignment between the representations of the two situations (Falkenhainer et al., 1989; Gentner & Markman, 1997; Markman & Gentner, 1993a; Medin, Goldstone, & Gentner, 1993). The mapping process is operationalized in the Structure Mapping Engine (SME; Falkenhainer et al., 1989; Forbus, Gentner, & Law, 1995). SME operates in a local-to-global fashion (see Fig. 1). It begins with a local, structurally blind free-for-all: All possible matches between individual elements of the two representations are found (resulting in many inconsistencies). In stage 2, these matches are combined into several structurally consistent set) are combined into an overall mapping, with the largest and most deeply connected kernels being favored in the merge process (again, the systematicity principle). The resulting alignment consists of an explicit set of



Fig. 1. Overview of SME (the Structure-mapping Engine) showing how an initially blind, local-to-global process produces structurally consistent inferences.

correspondences between the elements of the two representations. As a natural outcome of the alignment process, candidate inferences are projected from the base to the target. These inferences are propositions connected to the common system in one analog, but not yet present in the other.

The alignment process is guided by a set of tacit constraints that lead to structural consistency: (a) there must be *one-to-one correspondence* between the mapped elements in the target and base, and (b) there must be *parallel connectivity* (i.e., if two predicates correspond, then their arguments must correspond as well). A further assumption is the *systematicity principle:* A bias to prefer interpretations in which the lower-order matches (such as events) are connected by higher-order constraining relations (such as causal relations). The systematicity principle stems from a tacit preference for coherence and predictive power. Thus, when a given analogy affords more than one consistent interpretation, people prefer the more systematic interpretation, all else being equal. Further, if given two examples to compare, people tend to choose the one with deeper causal or explanatory structure as the base domain, and use it to structure the less systematic case (Bowdle & Gentner, 1997).

Achieving a structural alignment sets the stage for four kinds of learning:³ abstraction, contrast, inference-projection, and rerepresentation. In *abstraction*, the common system resulting from the alignment becomes more salient and more available for future use (Fig. 2a). In *contrast*, alignable differences—differences that occupy the same role in the two systems—are highlighted (Fig. 2a). *Inference-projection* occurs when one member of the pair is more complete in its structure than the other; in this case, spontaneous *candidate inferences* will be made that enrich the less-complete item (Fig. 2b). A further way that learning can occur is *rerepresentation*: If there is good reason to believe two (nonidentical) relations should match (e.g., a very good overall structural match), then one or both of the



Fig. 2. Comparison as structure-mapping. (A) Structural alignment highlights common structure (which may then serve as an abstraction) and alignable differences. (B) Structural alignment supports the projection of candidate inferences.

nonmatching predicates may be rerepresented to permit the overall match. This paper focuses chiefly on the first three processes.

This basic structure-mapping framework is now widely shared among analogy researchers (Doumas, Hummel, & Sandhofer, 2008; French, 2002; Gentner, Holyoak, & Kokinov, 2001; Holyoak & Thagard, 1995; Hummel & Holyoak, 1997; Kokinov & Petrov, 2001; Larkey & Love, 2003; Ramscar & Yarlett, 2003; see Gentner & Farbus, in press, and Kokinov & French, 2002, for reviews). However, accounts differ as to *how* the process takes place—an issue crucial to understanding the role of analogical processing in development. There are three aspects of SME that make it particularly apt for modeling cognitive developmental bootstrapping.

• In SME, the initial process is a symmetric structural alignment between the two representations. New inferences and alignable differences follow from this alignment. In contrast, many other models of analogical processing begin by choosing a structure in the base and projecting it to the target. This is an appealing intuition, and indeed it was my original proposal in the early 80s (e.g., Gentner, 1983); but it has a big drawback for development: It requires a well-structured base analog, in which the key structure is clear, in order for experiential learning to take place. Such a process could not account for the speed of learning in young children and infants, who have not yet accumulated a store of clearly structured examples. Moreover, research shows that young children *can* gain from comparing things even when neither of them is yet well understood (Gentner & Namy, 1999; Kotovsky & Gentner, 1996; Son, Smith, & Goldstone, 2007). SME's process of structural alignment can capture the fact that a learner who aligns two partially understood exemplars can still gain insight from

deriving the common relational structure.⁴ As Namy and Gentner (2002) put it, *two* sow's ears—if properly aligned—can yield a silk purse.

- A related advantage is that SME does not require the learner to know (or even guess) what the point of the comparison will be in advance. The process begins blind and local, but the final alignment may reveal a new abstraction, highlight a key difference, or prompt a spontaneous new inference. This feature is crucial for its role as a bootstrapping process in cognitive development. Essentially, it allows a young learner to stumble into insight simply by having the natural curiosity to compare two things.
- In SME, the same process operates for literal similarity as for analogy. In both cases, the initial local match stage finds all matches at every level, from object attributes to higher order relations, and further processes arrive at a global match based on structural consistency and systematicity. The difference between literal similarity and analogy is simply in the nature of the winning interpretation: If the final common system is purely relational, then the comparison is an analogy; if the common system is richer, containing many object attributes as well as relations, then it is a literal similarity match. This allows SME to capture the finding that progressive alignment from close to far matches can allow children to progress rapidly, as discussed below.

To reprise, there are at least four ways in which structural alignment furthers the acquisition of knowledge.

- 1. *Highlighting and schema abstraction*: Structural alignment results in extracting a common system from the two representations, thereby promoting the disembedding of hitherto implicit common structure, especially common relational systems (Gentner, 1983; Gick & Holyoak, 1983; Markman & Gentner, 1993a; Namy & Gentner, 2002).
- 2. *Rerepresentation*: If there is good reason to believe two (nonidentical) relations should match—for example, if an instructor has provided the analogy, or if the rest of the structure aligns well—then one or both of the nonmatching predicates may be rerepresented to permit the overall match (Forbus et al., 1995; Son et al., 2007; Yan, Forbus, & Gentner, 2003). Typically, this is done by minimal ascension (Falkenhainer et al., 1989): that is, by finding the most specific relation that is superordinate to the two nonmatching relations. Rerepresentation promotes *uniform relational encoding*, which (as discussed later) is crucial to expertise.
- 3. *Inference-projection*: Structural alignment leads to candidate inferences that enrich one or the other representation (Bowdle & Gentner, 1997; Clement & Gentner, 1991; Markman, 1997).
- 4. *Difference-detection*: Structural alignment leads naturally to the highlighting of alignable differences, fostering learning by *contrast* (Gentner & Markman, 1994; Gentner & Sagi, 2006; Markman & Gentner, 1993b; Ming, 2009).

I next discuss how structural alignment operates in children's learning, focusing first on highlighting and abstraction, and then on difference-detection. Next, I discuss the contribution of language to this processing. Finally, I consider the joint operation of language and structure-mapping in the development of numerical cognition.

3. Highlighting common relational structure

An important implication of the process model outlined above is that the simple act of comparing two things promotes a structural alignment that renders the common structure more salient (Gentner & Wolff, 1997; Gick & Holyoak, 1983; Markman & Gentner, 1993a). For example, adults list more commonalities when comparing pairs of similar concepts than when comparing pairs of dissimilar concepts (Markman & Gentner, 1993b; Tversky, 1977), and the same is true for children (Gelman, Raman, & Gentner, 2009). Comparison favors noticing commonalities and increases the pair's perceived similarity, and this is especially true for children (Hammer, Diesendruck, Weinshall, & Hochstein, 2009). Boroditsky (2007) found that comparison increased subjective similarity even when participants were asked to state differences. Comparison has also been shown to aid young children's learning of categories and schemas (Gentner & Namy, 1999; Hammer et al., 2009; Liu, Golinkoff, & Sak, 2001; Namy & Gentner, 2002) and of conceptual regularities (Chen & Klahr, 1999; Gentner, Levine, Dhillon, & Poltermann, 2009; Kotovsky & Gentner, 1996; Opfer & Siegler, 2007; Thompson & Opfer, in press), even in infants (Oakes & Ribar, 2005; Wang & Baillargeon, 2008).

Of course, a general highlighting of commonalities would be predicted by many theories of comparison, including Tversky's (1977) independent-features theory. The more telling point is that connected relational structure is preferentially highlighted during comparison. For example, adults asked to match elements between two pictures are more likely to choose correspondences based on common relational role (rather than matching similar objects) if they have previously compared the two pictures (Markman & Gentner, 1993a). A particularly convincing line of support for the claim that comparison preferentially highlights common relational structure comes from studies by Gentner and Namy (1999; Namy & Gentner, 2002). In these studies, 4-year-old children were given a novel label (e.g., dax) for a pictured object (e.g., a bicycle) and asked to choose another dax from two alternatives: a perceptually similar (same shape) object from a different category (e.g., eyeglasses) or a perceptually dissimilar match from the same high-level category (e.g., a skateboard-another vehicle). When children saw either standard (bicycle or tricycle) singly, they tended to choose the perceptual match, consistent with prior studies showing a shape bias in children's word learning (Baldwin, 1989; Imai, Gentner, & Uchida, 1994; Landau, Smith, & Jones, 1988). However, children who compared the two standards (e.g., bicycle and tricycle) were significantly more likely to choose the relational match. This result cannot be predicted by a traditional account of similarity, in which comparison is viewed simply as an intersection of the properties of the two items. Because the two standards always shared the same salient perceptual properties with each other as they did with the perceptual alternative, such an intersection process would have *heightened* the salience of the common perceptual properties and led to more perceptual responding rather than less. Instead, it appears that comparison selectively favored relational commonalities such as "both can be ridden" and "both are used outside"—consistent with the idea that comparison induces a focus on common relational structure.

Further evidence that comparison highlights relational commonalties comes from research in transfer of learning. Comparing two scenarios dramatically increases the likelihood that a principle common to two exemplars will be transferred to a future item (relative to seeing just one exemplar, or even the same two items without encouragement to compare) (Catrambone & Holyoak, 1989; Gentner, Anggoro, & Klibanoff, in press; Gick & Holyoak, 1983). For example, Loewenstein, Thompson, and Gentner (1999) found that business school students who compared two negotiation scenarios were over twice as likely to transfer the negotiation strategy to an analogous test negotiation as were those who studied the same two scenarios separately.

3.1. Structural alignment supports learning new relational patterns

Taking advantage of this highlighting effect, Stella Christie and I have used comparison to teach children novel spatial configurations (Christie & Gentner, in press). Christie and Gentner used the Comparison-versus-Solo word-extension task developed by Gentner and Namy (1999) to test whether structural alignment would help young children learn novel spatial configurations, such as "small thing above big thing, otherwise identical" (Fig. 3).



Example Stimulus Set

Fig. 3. Sample set from Christie and Gentner (in press). In the Solo condition, children saw one of the standards and were given a novel label (e.g., *blicket*) for it. In the Comparison condition, children saw both standards together, were told they were both blickets, and were encouraged to compare them. Both groups were then asked which alternative was also a blicket.

Children (3- and 4-year-olds) were told the name (say, *blicket*) of either one standard (Solo condition) or two (Comparison condition). In the Comparison condition, children were asked, "Do you see why these are both blickets?" in order to encourage comparison of the standards. Then children were asked which of two alternatives was a blicket. One alternative (the Relational choice) depicted two new objects in the same configuration; the other shared an exact object match with the standard (or two object matches, in the Comparison condition).

We expected this to be a challenging task, because the configurations were unfamiliar and because we know from prior research that object matches are extremely appealing to young children (Chen, 1996; Gentner, 1988; Gentner & Toupin, 1986; Richland et al., 2006). Indeed, in the Solo condition, 3-year-olds chose the relational match only 2% of the time (i.e., they chose 98% object matches). In sharp contrast, 3-year-olds who compared two standards chose the relational alternative 57% of the time. Importantly, a further study showed that this increase in relational responding did not occur when children saw the same two standards sequentially. Even when the standards were presented and named in immediate sequence, children's word-extension performance did not differ significantly from that in the Solo condition. Simply hearing that both standards were blickets was not enough; children had to compare them to experience the relational insight. This leads to the suggestion that cross-situational learning may succeed in just those cases when alignment naturally occurs–such as when the items are highly similar or are ordered in a progressive alignment sequence, as described next.

3.2. Progressive alignment and experiential learning

Highlighting of common structure occurs to some degree for close matches as well as for "far" analogical matches. This means that even close matches can bootstrap learning in young children (Claim 3 of the initial claims). Carrying out a concrete literal similarity match involves an alignment of relational structure, just as does analogy—the difference is that literal similarity also involves object matches. Such literal matches are a major learning route for young children, for two reasons: (a) their high overall similarity makes it likely that children will spontaneously compare them; and (b) because the object matches support the relational matches, even young children are likely to arrive at the correct alignment. Close matches thus serve as "training wheels" for more challenging, purely relational matches (Gentner et al., in press). The following three studies illustrate this phenomenon of progressive alignment, whereby carrying out an easy literal match confers the beginnings of relational insight (Gentner & Medina, 1998; Kotovsky & Gentner, 1996):

Loewenstein and Gentner (2001) gave children (aged 3¹/₂) a challenging search task modeled after DeLoache's (1987) classic studies. Children watched the experimenter hide a toy in a small model room (the Hiding room), and then tried to find the toy hidden "in the same place" in a second model room (the Finding room). The Finding room contained the same type of furniture (bed, table, etc.) as the Hiding room, in the same configuration, but was dissimilar in the specific shapes of its furniture, making

the mapping task difficult for young children (Blades & Cooke, 1994; DeLoache, 1987). Before engaging in the task, all the children were shown the Hiding room along with another highly similar room (identical except for color). Half the children saw the two rooms together and were encouraged to compare them; the other half discussed each room separately. Children who had compared the rooms were significantly more likely to correctly locate the toy in the Finding room than those who had experienced the rooms separately. Thus, comparing two nearly identical rooms facilitated children's ability to map their common spatial relational structure to a relationally similar but surface-dissimilar target.

- Haryu, Imai, and Okada (in press) taught 4-year-old children a verb for a novel event ٠ and asked whether the children could extend the verb to other events. They found that children were initially limited to close overall matches (i.e., literally similar events): That is, they extended the verb only when the new event shared objects as well as action with the initial event, and failed when the new event shared only its action with the initial event. In a second study, Haryu et al. found that progressive alignment from close to far matches enabled a new group of 4-year-olds to extend the verb based on sameness of action, without support from object similarity. As in the Loewenstein and Gentner study just described, high object similarity led children to make the correct correspondences, which supported the correct structural alignment. Achieving this alignment resulted in heightening the salience of the relational structure, which the children could then extend to an event that shared *only* that structure. These findings are consistent with the position that verb meanings are bootstrapped by comparing utterances in which verbs appear in very similar frames (Childers, 2005; Childers & Paik, 2009; Childers & Tomasello, 2001; Pruden, Hirsh-Pasek, Shallcross, & Golinkoff, 2008; Piccin & Waxman, 2007; see Gentner & Namy, 2006 for longer discussion).
- Thompson and Opfer (in press) used progressive alignment to help second-grade children overcome the logarithmic spacing tendency (whereby children represent number lines with wider spacing between smaller numbers than between larger numbers). These children had correct linear representations for small numbers (0–10 and 0–100) but made logarithmic scales for 0–1,000 and above (Siegler & Opfer, 2003). In the progressive alignment condition, children first compared different manifestations of the 0–100 scale, so they could see that a given number—say, 15—appears in the same place. Then they gave the child a series of pairwise comparisons with larger scales: For example, placing the number 15 on the 0–100 line and placing the number 1,500 on the 1–10,000 line. In the posttest, children who had received this progressive alignment training showed linear representations on all three scales, outperforming children who had received all the same exemplars separately.

In sum, progressive alignment can foster rapid learning. Early in learning, children are unlikely to spontaneously discover far analogies, but they naturally compare closely similar pairs that occur in close proximity (Kotovsky & Gentner, 1996). These close, highly alignable matches can bootstrap young children to a more distant relational mapping. Progressive alignment offers a route by which children's ordinary experiential learning can gradually lead them to the discovery of analogical matches (Gentner & Medina, 1998). Indeed, I suspect that for young children seemingly mundane repetition offers moments of invisible learning that seed further abstractions.

3.3. Structural alignment promotes abstraction in infants

A particularly dramatic example of early learning was found by Marcus, Vijayan, Rao, and Vishton (1999), using simple language-like stimuli. They found that 7-month-old infants, after repeated exposure to like patterns of syllables, could learn to recognize that same pattern even with new syllables. For example, if the infants had heard several instances of an ABA pattern, and then were presented with new syllables in either the same ABA pattern or a new pattern (say, ABB), they showed a novelty response to the new pattern. Kuehne, Gentner, and Forbus (2000) simulated this "infant rule-learning" using a model of learning by progressive alignment. This model, called SEQL (Kuehne, Forbus, Gentner, & Quinn, 2000), forms abstractions across a set of exemplars by making successive structural comparisons among exemplars (using SME).⁵ When a new exemplar is introduced, it is compared to the existing abstractions and (if sufficiently similar) assimilated into that abstraction, typically resulting in a slightly more abstract generalization. Exemplars that cannot be assimilated into any existing category (because they are too dissimilar from the existing generalizations) are maintained as separate exemplars. New exemplars are compared first to existing abstractions, then to stored exemplars. If none of these exceeds the similarity threshold for combining representations, then the new item is stored as a separate exemplar.

The SEQL simulation was able to learn the Marcus et al. language-like patterns within the same number of trials as the infants, without feedback, and without pretraining, in contrast to connectionist simulations of the same phenomenon, which typically require extensive pretraining and/or many thousands of learning trials. Two aspects of SEQL's performance seem apt in capturing the infants' novelty response. First, when presented with new strings, it found those with different structure far less similar (more novel) than those with the same structure. Second, when confronted with a new structure, it made incorrect candidate inferences: For example, if it had received ABA study items and was given an ABB test exemplar, it used its ABA schema to infer that the third syllable in the test item should be an A—a prediction that was immediately contradicted. This contradicted inference, we suggest, corresponds to a violation of expectations.

Interestingly, although the simulation matched the infant response pattern quite closely, it did not do so on the basis of a fully abstract rule. On every run, the generalization retained some surface features. Yet because of the structural character of the matching process, SEQL still found new instances with matching structure to be much more similar than those with different structure. These findings raise the possibility that some of the seemingly abstract rules of grammar may in fact be simply near-abstractions resulting from progressive alignment.

4. Structural alignment supports learning by contrast

Another way that structural alignment promotes learning is that it highlights alignable differences (Markman & Gentner, 1993b). When two things are compared, differences connected to their common structure stand forth to the learner, making these differences available for learning by contrast (Gentner, Loewenstein, & Hung, 2007; Ming, 2009). For example, Gelman et al. (2009) asked 4-year-olds and adults to generate commonalities or differences for pairs of pictured objects. Both age groups generated both more commonalities *and* more alignable differences for high-similarity pairs (e.g., cat/dog) than for low-similarity pairs (e.g., cat/fork). This is consistent with the claim that structural alignment potentiates noticing commonalities and alignable differences; since high-similarity pairs yield larger aligned structures (as well as being easier to align) (Lovett et al., 2009), they should generate more commonalities and more differences than low-similarity pairs. Interestingly, children who heard generic wording (Gelman, 2003) generated deeper commonalities and differences than did those who heard specific language.

Evidence that structural alignment can reveal alignable differences, and that this process can aid children's learning, comes from a study conducted at the Chicago Children's Museum (Gentner et al., 2009). The main goal of the exhibit was for children and their families to engage in a free-form construction of a model skyscraper together. Our instructional opportunity was extremely brief session prior to this activity. Yet by using the principles of alignment and contrast, we were able to achieve a fair degree of learning in roughly 2 min per child.

Our goal was to convey a key principle of stable construction—namely, the idea of using a diagonal brace to stabilize a structure.⁶ This insight is typically not obvious to children, as Haden and colleagues have shown (Wilkerson, Benjamin, & Haden, 2007; see also Olson, 1970). Our method utilized three principles from structure-mapping theory, already reviewed: (a) *abstraction*: analogical comparison reveals common structure; (b) *contrast*: analogical comparison highlights alignable differences—differences along a common dimension or predicate that plays the same role in the common structure; and (c) *progressive alignment*: alignment is easier and less error-prone when the items being compared are highly similar in their objects and parts as well as in their relational structure; and such alignments potentiate further, more challenging alignments.

Children aged 6–8 years were randomly assigned to either a High Alignability, a Low Alignability, or a No Training condition. In the first two conditions, children were taken aside from their families just prior to the construction session and shown a pair of buildings. One building included a diagonal brace (which gave the structure stability) and the other had a horizontal crosspiece (which provided no structural support). Children were asked "Which one is stronger," and then (briefly) allowed to wiggle them; this revealed that the diagonally braced building was hard to distort, whereas the other could be greatly distorted. When asked again after the demonstration, all but a few children chose the braced building as strongest.

In the High Alignability condition, the two buildings differed only in this key feature of brace placement and were readily alignable otherwise. In the Low Alignability condition, the buildings were perceptually different—one was wider than the other—and were therefore harder to align. Although the Low Alignability pair had the same key difference as the High Alignability pair (a diagonal brace vs. a horizontal crosspiece), we predicted that this difference was more likely to be noticed in the High Alignability condition. The third group (the No training group) simply proceeded directly to the construction session.

After the roughly 13-min unsupervised construction activity, children were taken aside for a brief transfer test. They were shown a partly constructed, unstable building. The experimenter wiggled the building to show that it was not stable and then asked the child to place a beam so as to improve it "so that it doesn't wobble." The dependent measure was whether the child indicated a diagonal placement (correct) or a horizontal/vertical placement. The results showed a large effect of training: There were more diagonal placements among children who had received comparison training than among those who had not. More interestingly, children in the High Alignability condition (M = 0.65) were more likely to make a diagonal placement than those in the Low Alignability condition (M = 0.48) evidence that structural alignment is crucial to identifying differences as well as commonalities.

These findings show that even a single brief comparison experience can confer insight. They also add to evidence that the key to this insight is structural alignment. Children who received highly alignable models were more likely to derive the insight than those who did not. Thus, alignment is important not only for abstracting commonalities but also for detecting structurally relevant differences.

5. Language amplifies analogical processing to facilitate cognitive development

So far the discussion has centered on how structural alignment fosters learning. We now turn to how language interacts with comparison processes in learning and development. In this account, I adopt the *language as toolkit* view (Gentner, 2003; Gentner & Goldin-Meadow, 2003; see also Frank, Everett, Fedorenko, & Gibson, 2008), rather than a strong linguistic determinism view. The toolkit view holds that acquiring a language provides new resources that augment human cognitive capacities, but it does not replace prelinguistic abilities. This view is consistent with Carey's (1985a) proposal that language provides "tools of wide application" that facilitate forming particular representations and carrying out particular processes.

My colleagues and I have proposed four ways in which language interacts with analogical processes to foster learning and development (Gentner, 2003; Gentner & Christie, in press; Gentner & Namy, 2006; Loewenstein & Gentner, 2005).

- 1. *Common labels invite comparison and abstraction*: By giving two things the same name, we invite children to compare them; Gentner and Medina (1998) termed this *symbolic juxtaposition*.
- 2. *Naming promotes reification:* A linguistic label helps to preserve the abstraction derived from a comparison and to render it more accessible for future use

(Gentner, 2003; Gentner et al., in press; Lupyan, Rakison, & McClelland, 2007; Son & Goldstone, 2005).

- 3. *Naming promotes uniform relational encoding:* An important consequence of reification is the possibility of uniform relational encoding. Habitual use of a given relational term makes it more likely that the relational constellation will be encoded in the same manner at different times and in different contexts. This is important, if we assume (a) that memory retrieval is based on matching the current contents of working memory with a stored representation and (b) that one reason relational transfer is generally quite poor (compared to retrieval based on surface matches) is that relations are encoded more variably than objects (Forbus et al., 1995; Gentner et al., 2009). Uniform relational encoding is a major route to reliable relational retrieval.
- 4. *Linguistic structure invites conceptual structure*: Systematic structure in the language can invite correspondingly systematic conceptual structure (Gentner, 2003; Loewenstein & Gentner, 2005).

To see how these principles operate in concert with analogical processes, we turn to a specific example—the development of number.⁷ To set the stage, I first briefly review the acquisition of the natural numbers and the evidence for language effects in number development.

5.1. Language and the natural numbers

Intuitively, it seems that mathematical structure is so compelling that all humans must inevitably recognize at least the natural numbers. As the English mathematician G. H. Hardy stated, "I believe that mathematical reality lies outside us, that our function is to discover or observe it, and that the theorems which we prove, and which we describe grandiloquently as our 'creations,' are simply our notes of our observations" (quoted in Dehaene, 1997, p. 242). Yet there is evidence that even simple numerical insight is not inevitable, and that human language is instrumental in developing the very notion of number. For example, Spelke and Tsivkin (2001) trained Russian-English bilinguals to solve either exact calculation problems or approximation problems (which do not depend on exact number). When trained in one language, they readily transferred the approximation techniques to the other language; but when trained on exact calculation, their performance deteriorated sharply when asked to do similar problems in their other language. This pattern led Spelke and Tsivkin to suggest that language is implicated in the representation of exact large number.

How does this unfold developmentally? Two preverbal capacities enter into theories of numerical cognition: an analog magnitude system and a system for keeping track of small numbers of items. The analog magnitude system, shared broadly with other species, allows approximate judgments of quantity and is often modeled with an accumulator model (Gallistel & Gelman, 1992; Meck & Church, 1983). It is generally agreed to be language independent. The analog magnitude system operates over even very large quantities, but its accuracy is limited by Weber's law: The discriminability between two amounts is a function of their ratio. Thus, inaccuracies occur for large magnitudes that are very close. The other

system is a capacity for keeping track of small numbers of items—part of our general capacity for representing mental models of the world (e.g., Carey, 2004; Spelke, 2003). In contrast to the analog magnitude system, the object tracking system operates over discrete representations and is capacity limited, to roughly three or four objects.

Dramatic evidence that number language augments these preverbal capacities comes from studies of two Amazonian peoples whose languages-Pirahã and Mundurukú-lack a full counting system (Everett, 2005; Frank et al., 2008; Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004). In a ground-breaking study, Gordon (2004) investigated the Pirahã, whose language for numerical quantities appears to be "few, some, many.⁸ "When Gordon gave a variety of numerical tasks to Pirahã participants, he found striking failures even on what seem to be very simple tasks. For example, he showed participants a number of small objects in a line and asked them to make a line of batteries (a familiar object) that was similar but orthogonal to the experimenter's line. They were fairly accurate for amounts of three or fewer, but their performance became progressively more inaccurate as the number of objects increased. Similar results were obtained for the "nuts in a can" task, in which Gordon showed the participant an array of nuts, then put the nuts in a can and took them out one by one, each time asking participants whether there were still more nuts in the can. Again performance became more inaccurate the larger the number-the signature of the analog magnitude system. Frank et al. (2008) replicated Gordon's results with the Pirahã and confirmed that for many of Gordon's tasks⁹ the size of the error fit Weber's law.

This basic pattern was replicated by Pica et al. (2004), working with the Mundurukú. All three of these studies found greater inaccuracies for larger numbers, suggesting reliance on a signature of the analog magnitude system. Pica et al. further found that Mundurukú speakers were fairly comparable to French speakers on quantity estimation tasks, but greatly deficient on tasks that require exact computation beyond two or three. It appears that language profoundly influences numerical cognition, consistent with the cognitive toolkit account. However, again consistent with the toolkit account, language does not replace the preverbal capacity for magnitude estimation.

5.2. Acquiring the natural numbers

Bertrand Russell (1920) memorably stated: "It must have required many ages to discover that a brace of pheasants and a couple of days were both instances of the number 2: the degree of abstraction involved is far from easy." The aptness of this intuition can be seen in the difficulty children have in coming to understand the natural numbers. Children first learn the count routine as a kind of language game, with only a vague connection to numbers (Fuson, 1988). At this stage, a child may be fairly proficient at counting from 1 to 10, while at the same time showing little or no insight into cardinality. If a typical young 2 year old is asked, "How many marbles are there?" she will count the marbles; but having counted correctly to four, she is unable to report that there are four marbles. If pressed to say how many, she may simply execute the count routine again. Likewise, if asked to "show me three marbles" a young 2 year old will produce three, five, or eight marbles (Fuson, 1988; Wynn, 1992).

Gradually children learn to bind the numerals in the count routine to actual numerical quantities, beginning with very small setsizes. Children's first forays into the mapping between number and set-size are highly context specific (Mix, 2002; Mix, Sandhofer, & Baroody, 2005), as is often the case in early learning (Gentner & Medina, 1998). For example, Mix (2002) noted that at 20 months her son Spencer could reliably bring from another room exactly two treats for the family's two dogs; but when asked to go get "train treats" for his two toy trains, he failed. His understanding of "twoness" was highly context-bound, just as Russell might have predicted. However, children whose language contains a count list do not have to discover this abstraction by themselves; they are given a royal road to abstraction when they learn the count terms. As Mix et al. (2005) suggest, hearing different sets labeled with the same count word—say, "two"—prompts the child to compare the sets and to notice their common setsize. This is consistent with our proposal that common labels invite comparison and abstraction (Gentner & Medina, 1998).

5.3. Ordinal language invites ordinal numerical structure

So far we have considered how language interacts with relational processes to foster comparison and abstraction. We now turn to the fourth kind of interaction: that linguistic structure invites conceptual structure. We take up Susan Carey's proposal that an analogical mapping of ordinal structure from number language to quantity is instrumental in bootstrapping the cardinality principle—perhaps the clearest example of the power of systematic language to confer systematic conceptual structure.¹⁰

Carey's (2004, 2009) account agrees with those discussed above in assuming that number language is initially learned as a routine, well before the young children understand the exact mapping to particular quantities. Gradually, the child learns to attach number words to very small set sizes, which can be held by the object-tracking system. The learning is at first slow and piecemeal—even after binding *two* to sets of cardinality two, weeks or months may ensue before the child realizes that *three* refers to a set with three items (Carey, 2004; Mix, 2002; Mix et al., 2005). But once a child reaches an understanding of roughly *three* or *four*, the pattern changes; the child rapidly binds the succeeding numbers to their cardinalities. At this point, Carey suggests, the child has begun to see the analogy between the increase in the count sequence and the increase in set size.

The binding of *three* to the quantity three is a momentous connection for the parallelincrease analogy. At that point, as shown in Fig. 4, the child has two examples of coordination between *further-by-one* in count and *greater-by-one* in set size. Having noticed that this pattern of *parallel increase* holds for $1 \rightarrow 2$ and $2 \rightarrow 3$. the child wonders whether it will continue to hold. Lo and behold, it does: Counting on one more from "3" gives "4," which indeed corresponds to the set size one greater than {3}, and so on for larger and larger numbers. Eventually, I suggest, the two parallel relations are rerepresented as SUCCESSOR relations applying to different aspects of number: that is, SUCCESSOR[(count(n), count(n+1)] and SUCCESSOR[(setsize(n), setsize(n+1)]. At this point the analogy has yielded an extended relational abstraction.

766



Fig. 4. The analogy linking count sequence and numerical order [based on Carey's (2001, 2004, 2009) proposal]. (A) When the child has "2" connected to set size 2, and "3" to set size 3, this makes two instances of the same relational pattern—permitting an analogy. This analogy invites the inference (not shown here) that the same relational pattern will hold for "4": that is, that its set size will be one greater than the set size of "3." (B) The analogy also invites the abstraction IMPLIES {FURTHER-BY-ONE (count list) \rightarrow GREATER-BY-ONE (setsize)}, suggesting that the sequence continues indefinitely.

6. Structure-mapping in language acquisition

I have focused here on ways in which language acquisition contributes to analogical ability. However, the reverse interaction is equally important. Structure-mapping is a central mechanism by which humans acquire language (Gentner & Namy, 2006). There is evidence that comparison contributes to learning word meanings for nouns (Gentner & Namy, 1999, 2004; Liu et al., 2001), adjectives (Sandhofer & Smith, 2001; Waxman & Klibanoff, 2000), prepositions (Casasola, 2005), verbs (Childers, 2005; Childers & Paik, in press; Haryu et al., in press; Piccin & Waxman, 2007; Pruden et al., 2008), and even mental concepts (Baldwin & Saylor, 2005). There is also evidence that structure-mapping contributes to the acquisition of grammar (Casenhiser & Goldberg, 2005; Fischer, 1996; Kuehne, Gentner et al., 2000; Marcus et al., 1999 Tomasello, 2000). All this suggests a highly productive mutual causation between analogy and language.

7. General discussion

The twin mysteries of "Why we're so smart" and "How do children progress so fast?" are inextricably connected. My answer to both is the mutual facilitation between relational

ability and language. I began this paper by reviewing evidence that analogical processes are fundamental to human cognition and learning. I argued that comparison engages a process of structure-mapping that begins with structural alignment, and that achieving a structural alignment potentiates learning in at least four interrelated ways: (a) *abstraction*: the common system resulting from the alignment becomes more salient and more available for future use; (b) *rerepresentation* can occur, generally to a more abstract relation that captures both analogs; (c) *inference-projection*: spontaneous candidate inferences are made from a well-structured representation to one that is less complete; and (d) *difference-detection*: alignable differences—differences that occupy the same role in the two systems—are highlighted, fostering learning by contrast. I reviewed evidence for these kinds of comparison-driven learning in children's development. I also reviewed a process model, SME, which shows how new learning can be bootstrapped from initially partial understandings.

The second great force in our intellectual development is language. Although humans are born with high relational ability, interaction with language is necessary to realize the full potential of our analogical capacity (Gentner, 2003; Gentner & Christie, 2008; Gentner & Rattermann, 1991). Language—especially relational language—forms a positive feedback system with relational cognition that bootstraps human learning. This position is broadly consistent with Susan Carey's (2009) account of how Quinean bootstrapping supports conceptual change in children. I discussed four ways in which language can interact with analogical processing: (a) common labels invite comparison and abstraction between the things named; (b) labeling a concept (including one derived via comparison) reifies it, giving it more stability; (c) naming a concept also promotes uniform relational encoding, which increases the probability of relational retrieval; and (d) linguistic structure can invite corresponding conceptual structure. I reviewed several kinds of evidence for this claim that language bootstraps analogical processing and thereby promotes children's learning.

7.1. Some challenges

Before concluding this paper, I want to deal with some potential challenges. First, is structure-mapping claimed to be the only important human learning process? The answer is no; for example, associative learning and reinforcement learning are extremely important for humans, though widely shared with other species. Analogy also cannot take the place of higher level processes like causal reasoning, means-ends analysis, or logical deduction. However, I suggest that analogical processes are important in acquiring the relational knowledge that supports such reasoning. People often *derive* a causal model by analogy with another example, even though the actual process of causal reasoning is separate from the process of analogical mapping (Colhoun & Gentner, 2009).

The second challenge is whether it is relational ability in particular that distinguishes humans from other species. Studies of our closest relatives, chimpanzees, have shown that they possess some degree of relational ability (Haun & Call, 2009). However, a recent wide-ranging review concluded that the common denominator in tasks in which chimpanzees fall severely behind humans is the need for relational representation and matching (Penn et al., 2008). A telling point is that although chimpanzees can learn and use numbers

up to eight or nine (Boysen & Berntson, 1989), there is no evidence that they ever experience the analogical insight that comes to preschool children—that is, that the parallel-increase pattern occurs over and over.¹¹

Studies of chimpanzees are also relevant to the other half of the present thesis—the claim that relational language contributes to analogical ability. There is evidence that language-trained chimpanzees show considerably greater relational insight than those without language training (Oden, Thompson & Premack, 2001; Premack, 1983; see Gentner & Christie, 2008; Gentner & Rattermann, 1991).

The third challenge is to explain why, given this powerful mechanism, children do not learn even faster. One major reason is that children fail to notice many potential comparisons. Children are likely to miss potential analogies if they lack sufficient relational knowledge to align the analogs, or if the potential analogs are not highly surface-similar or do not occur in close juxtaposition. Even adults often fail to retrieve potential analogs, and adults have the benefit of a large relational vocabulary (with the attendant benefit of uniform relational encoding) to aid in their relational retrieval. Thus, if two highly alignable events happen to be juxtaposed, the child can compare and abstract; but if they are not sufficiently similar, or not sufficiently close in time, then nothing will be gained. Interestingly, from this account it follows that sequences of close, highly alignable exemplars should be the ideal learning situation. Such a situation exists in many infant habituation studies—suggesting that these studies are about what babies can learn as well as about what they know.

8. Conclusion

The ability to represent and reason about relational structure lies at the core of human cognitive powers. Aligning and mapping on the basis of shared relational structure is a general learning process that allows young humans to form abstract ideas from ordinary experience, and this ability is massively amplified by language and culture.

Notes

- 1. A simpler form of comparison may occur for very simple stimuli such as color swatches.
- 2. The theory assumes structured representations in which the elements are connected by labeled relations, and higher order relations (such as causal relations) connect first-order statements (see Falkenhainer, Forbus, & Gentner, 1989; Markman, 1999).
- 3. Analogy is also a major contributor to another form of knowledge change, namely, *restructuring*—altering the domain structure of one domain in terms of the other. This is a true case of conceptual change in Carey's (1985b) sense. However, true restructuring almost certainly requires more other processes in addition to analogy (Dunbar, 1995).

- 4. The claim that comparison processing—whether of similarity, analogy or metaphor—is symmetric at the outset may seem far-fetched, given the strong directionality of analogy and metaphor (Ortony, 1979). However, there is evidence that comparison processes are symmetric in the initial stages of processing, even for highly directional comparisons (Gentner & Wolff, 1997; Wolff & Gentner, 2000, unpublished data).
- 5. The current version of this system is called SAGE.
- 6. The underlying principle here is that the triangle is the only stable polygon: That is, in a triangle, one cannot change the shape without changing the length of at least one of the sides.
- 7. There is also considerable evidence that acquiring language augments children's ability to represent and reason about space (Bowerman & Choi, 2003; Dessalegn & Landau, 2008; Hermer-Vasquez, Spelke, & Katsnelson, 1999; Loewenstein & Gentner, 2005) and other domains (see Bowerman & Levinson, 2001; Gentner & Goldin-Meadow, 2003). For discussions of how spatial language and spatial analogizing interact in the spatial domain, see S. Christie, D. Gentner, & S. Goldin-Meadow (unpublished data).
- 8. Although this system was originally thought to be a "one, two, many" system, Frank et al. (2008) found that when given the naming task in reverse (10–1) order, the Pirahã assigned these terms differently: For example, the putative "one" term (hoi) was used for one, two, or three items, and the putative "two" term (hoi) was used frequently for six, seven, or eight items.
- 9. Interestingly, both Gordon (2004) and Frank et al. (2008) found high accuracy even for larger numbers (8 or 9) when the task was to make a line parallel to the original line—possibly because this task can be accomplished by one-to-one correspondence and does not require a sense of exact number.
- 10. See Spelke (2003) for a related but distinct view of how language supports numerical cognition. For opposing views, see Rips, Bloomfield, and Asmuth (2008) and Gallistel and Gelman (2000).
- 11. It must be noted that this difference could stem from specific aspects of the way chimpanzees are taught language. Chimpanzees do not receive the extensive evidence (from counting routines, etc.) that inculcates the count sequence; and, whereas children typically hear numbers like "50," "100," and above, chimpanzees may not receive evidence that the count string goes on indefinitely.

Acknowledgments

This research was supported by NSF SLC grant SBE-0541957, the Spatial Intelligence and Learning Center (SILC). Development of the structure-mapping theory and simulation was supported by the Office of Naval Research, award number N00014-02-10078. I thank Stella Christie, Nina Simms, and others in the Cognition and Language Lab at Northwestern University for discussions of the ideas, and Benjamin Dionysus and Katherine James for technical assistance.

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