Gentner, D. (2002). Analogy in scientific discovery: The case of Johannes Kepler. In L. Magnani & N. J. Nersessian (Eds.), *Model-based reasoning: Science, technology, values* (pp.21-39). New York: Kluwer Academic/ Plenum Publisher.

Analogy in Scientific Discovery: The Case of Johannes Kepler

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Abstract: Analogy is often linked with scientific discovery. In this paper I lay out a theory of analogical processing and apply it to the work of Johannes Kepler. Kepler is striking not only for the magnitude of his discoveries but because of the articulateness with which he laid out his reasoning processes, including his frequent and extended use of analogies. I discuss four analogical subprocesses - highlighting common structure, projecting inferences, re-representing relations, and noticing alignable differences - by which analogy brings about new ideas and show how they apply in Kepler's work. I focus particularly on a central extended analogy in which he used the phenomenon of the *light* from the sun, which travels to the planets and illuminates them, as a base domain for a new ontological entity (a precursor of gravity) - the *vis motrix,* that causes the planets to move.

Analogical reasoning can lead to change of knowledge - not only to enrichment of existing representations but also to true conceptual change. The work of Johannes Kepler (1571-1630) offers a clear example of the use of analogical processes in discovery. Kepler seems to have been a natural analogist. He used analogies constantly, both to make local points and to frame global theories. I focus here on one central analogy that played a formative note in this thinking: an analogy between light from the sun that illuminates the planets to something else - initially a spirit and later a force or power - emanating from the sun that causes the planets to move. He conceived this idea in its initial form early in his career, and over the next 20 years, used the analogy with light to develop the idea further of a *vis motrix* (a motive power between the sun and planets). As Toulmin and Goodfield (1961, p. 198) put it, "The lifelong, self-appointed mission of Johann Kepler

[...] was to reveal the new, inner coherence of the Sun-centered planetary system. His central aim was to produce a 'celestial physics,' a system of astronomy of a new kind, in which the forces responsible for the phenomena were brought to light".

In this paper I will try to show how the cognitive processes inherent in analogy can promote conceptual change. I lay out a set of analogical mechanisms by which analogy can act to create changes in knowledge and consider how these analogical mechanisms could have brought about the kinds of theory change that Kepler experienced. I begin with a tour of Kepler's discoveries. Then I give a brief review of the slate of knowledge existing when Kepler began his work. Then I describe the evolution of Kepler's great analogy. Next comes a description of analogical processing followed by its application to Kepler. Finally, I consider the psychological status of Kepler's analogies.

Kepler's best-known discoveries are the three laws of planetary motion:

1. (1609) First Law: The planetary orbits are ellipses with the sun at our forces.
2. (1596, 1609) Second Law: A line between the sun and any planet sweeps out equal times.
3. (1634) Third Law: The square of the period of a planet's orbit is proportional to the cube of its radius: $T^2 = kR^3$

He made a number of other discoveries, including an incremental calculus for computing the volume of a wine barrel; the modern theory that comet's tails consist of ejected matter; the proposal that the moon causes tides; a detailed theory of optics; writings on the comic sections and, in 1627, the Rudolphine Tables for predicting planetary motions, which were roughly 30 times more accurate than prior tables (Gingerich, 1993, p. 50).

But by far Kepler's most important discovery was his causal theory of planetary motion. As Gingerich (1993) puts it, Kepler's most consequential achievement was the mechanizing and perfecting of the world system. By the *mechanization* of the solar system, I mean his insistence on a "new astronomy based on the causes, or the celestial physics", as he tells us in the title of his great book [the *Astronomia Nova*]. By the *perfection* of the planetary system, I mean the fantastic improvement of nearly two orders of magnitude in the prediction of planetary positions (p. 333).

To see the magnitude of Kepler's discovery, we begin by reviewing the system of cosmology that Kepler inherited when be began his work in 1590. The ontology and explanatory system of medieval astronomy followed a line laid down by Plato and Aristotle and culminating in Ptolemy's system of the II century AD:

1. The earth is at the center of the universe and is itself unmoving.
2. The earth is surrounded by physically real crystalline spheres, containing the heavenly bodies, which revolve around the Earth.
3. The heavenly bodies move in perfect circles at uniform velocity. (However, epicycles and eccentrically positioned circles were used to account for the observed motions).
4. Heavenly bodies and their spheres are composed not of the four terrestrial elements - earth, air, fire and water, but of a fifth element, the *quintessence* - a crystalline aether that is pure, unalterable, transparent, and weightless. The farther from Earth, the purer is the sphere. By the same token, celestial phenomena must be explained in different terms from earthly phenomena.
5. All motion requires a mover. In the Aristotelian universe, the outermost sphere, containing the fixed stars, is moved by an "unmoved mover," the *primum mobile*. Each sphere imparts motion to the next one in.
6. Celestial bodies have souls. In particular, each planet is controlled by its own spirit, which mediates its motion. (This last principle, descended from the Stoics, enjoyed a resurgence in the XVI century for reasons explained below).

When this Aristotelian-Ptolemaic system was integrated with Catholic theology in the early XIII century by Albertus Magnus and Thomas Aquinas, angelic spirits were assigned to the celestrial spheres in order of rank, from Seraphim in the outermost and purest sphere, that of the *primum mobile,* inward to Cherubim (controlling the sphere of fixed stars) Thrones, Dominations, Virtues, Powers, Principalities, Archangels, and finally Angels, (controlling the sphere of the moon). The resulting conceptual scheme, dominant until the XVI century, was one of extreme intricacy and cohesion.

The Stoic view overlapped with the Aristotelian-Ptolemaic view in being geocentric, with a sphere of fixed stars. However, it differed in that it postulated that the heavens were filled not with pure aether but with a kind of intelligent pneuma (a combination of fire and air), which became more pure with distance from the earth. The heavenly bodies, also made of pneuma, were intelligent and capable of self-direction (Barker, 1991). There were thus two explanations for celestial motion: (1) the transmission of motion through interlocking crystalline spheres, and (2) the action of planetary intelligences or souls. Kepler dismissed the first because of two recent astronomical events. The first was a new star - a nova (or supernova) in 1572 - evidence against the Aristotelian doctrine of the unchanging and incorruptible firmament. The second was a comet in 1577 (and others not long after), whose path ran through the planetary spheres - which should therefore have been cracked. This was evidence against the view that the planets moved by

attachment to crystalline spheres. This left the idea of guiding spirits moving the planets. Throughout his career Kepler wrestled with the idea that the planets move themselves intelligently; gradually paring away aspects of intelligent thought in search of a more purely physical description.

The other major event that set the stage was Nicolaus Copernicus's publication of *De revolutionibus orbium celestium* in 1543, the year of his death - thirteen centuries after Ptolemy's model and thirty years before Kepler's birth - proposing the idea that the earth and other planets moved around the sun. Copernicus argued for his system on grounds of mathematical elegance and sufficiency, not on the basis of physical causation. Indeed, for mathematical reasons, he placed the center of the solar system at the center of the Earth's orbit, rather than the center of the sun itself.

Kepler embraced Copernicus's heliocentric characterization of planetary orbits. But from the start, he changed this conception radically, infusing it with causal significance. This causal interpretation of Copernicus's theory led to a reaxiomitization of astronomy. As Gingerich (p. 333) notes, "Copernicus gave the world a revolutionary helio*static* system, but Kepler made it into a *heliocentric system.* In Kepler's universe, the Sun has a fundamental physically motivated centrality that is essentially lacking in *De revolutionibus.* We have grown so accustomed to calling this the Copernican system that we usually forget than many of its attributes could better be called the Keplerian system." In part this is Kepler's own doing: he saw himself as simply making clear the real significance of Copernicus's work and titled one of his great works *The Epitome of Copernican Astronomy.*

With this as background, in 1596 the 25-year-old Kepler posed a simple but profound question: Why do the outer planets move slower than the inner planets? He noticed that the periods of the outer planets were longer, relative to those of the inner planets, than could be predicted simply from the greater distances they had to travel - that is, they traveled slower. From within the Stoic cosmology of planetary spirits, he asked whether the "moving souls" were simply weaker the further the planet. Instead, he conjectured the planets might be moved not by their own individual spirits but one spirit residing in the sun - the *anima motrix.* In this conjecture, he drew on an analogy with the sun's light (Kepler, *Mysterium cosmographicum*, 1596, p. 199):

> [...] one of two conclusions must be reached: either the moving souls [*motricis animae*] are weaker the further they are from the Sun; or, there is a single moving soul [*motricem animam*] in the center of all the spheres, that is, in the Sun, and it impels each body more strongly in proportion to how near it is.

He reasons that just as light grows fainter with distance, so might this motivating spirit or power (Kepler, *Mysterium cosmographicum,* 1596, p. 201).

> Let us suppose, then, as is highly probable, that motion is dispensed by the Sun in the same proportion as light. Now the ratio in which light spreading out from a center is weakened is stated by the opticians. For the amount of light in a small circle is the same as the amount of light or of the solar rays in the great one. Hence, as it is more concentrated in the small circle, and more thinly spread in the great one, the measure of this thinning out must be sought in the actual ratio of the circles, both for light and for the moving power [*motrice virtute*] [...]

Kepler was well aware of the implausibilities in his proposal. In the *Astronomia Nova* (1609), Kepler challenged his theory with the thorny question of action at a distance:

> For it was said above that this motive power is extended throughout the space of the world, in some places more concentrated and in others more spread out...This implies that it is poured out throughout the whole world, and yet does not exist anywhere but where there is something movable (Kepler 1609/1992, p. 382).

He answers this challenge by invoking the light analogy:

> But lest I appear to philosophize with excessive insolence, I shall propose to the reader the clearly authentic example of light, since it also makes its nest in the sun, thence to break forth into the whole world as a companion to this motive power. Who, I ask, will say that light is something material? Nevertheless, it carries out its operations with respect to place, suffers alteration, is reflected and refracted, and assumes quantities so as to be dense or rare, and to be capable of being taken as a surface wherever it falls upon something illuminable. Now just as it is said in optics, that light does not exist in the intermediate space between the source and the illuminable, this is equally true of the motive power (Kepler, 1609/1992, p.383).

It can be seen here that already by 1609 the idea of a motive *spirit* in the sun was giving way to a motive *power* or *force*. By the time of his 1621 revision of the *Mysterium cosmographicum*, Kepler had fully re-represented the "soul" of the sun to be a physical force or power (Kepler, *Mysterium cosmographicum*, 1621, p. 201):

> If for the word "soul" [*Anima*] you substitute the word "force" [*Vim*], you have the very same principle on which the Celestial Physics is established [...] For once I believed that the cause which moves the planets

was precisely a soul, as I was of course imbued with the doctrines ... on moving intelligences. But when I pondered that this moving cause grows weaker with distance, and that the Sun's light also grows thinner with distance from the Sun, from that I concluded, that this force is something corporeal, that is, an emanation which a body emits, but an immaterial one.

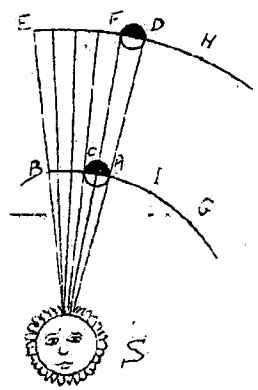

Figure 1. Kepler's depiction of the sun's light spreading outward.

Kepler returned repeatedly to the light-*vis motrix* analogy, analyzing and extending it across several works. From it he derived that (like light) the *vis motrix* travels invisibly from the sun to the planet through space and is detectable only when it contacts a planet. As with light, the concentration of the *vis motrix* should decrease with distance, this explains why the speed of the planet decreases with distance from the sun (analogous to the diminishing of brightness with distance from a light source). He delved into the base domain of this analogy - the behavior of light. He published a treatise on astronomical optics in 1604 (*Astronomiae pars optica*) and another in 1610 (the *Dioptrice*). With this considerable knowledge of the behavior of light, Kepler had a base domain that was systematic and well-understood - ideally suited to provide inferential resources for the target.

The *vis motrix* or *virtus motrix* (roughly, *force* or *power*) was a true precursor of gravity - a power that acts at a distance, in a constrained and lawful fashion. It was a fundamental change in the ontology of the solar system. Kepler did not arrive at the full notion of gravity; that was left for Newton some 80 years later; Kepler still believed that the sun's power pushed the planets around instead of attracting them inward. But he had something very close to an inverse square law for the strength of this central power, and he

applied this attractive power (like gravity) not only between the sun and planets, but between the earth and moon and tides.

The magnitude of the conceptual change that occurred over the course of Kepler's work is hard to grasp, in part because we now live in a cosmology defined by the changes brought by Kepler and his contemporaries (see Table 1). One major change Kepler's abandonment of the idea that the planets move in perfect circles at uniform speed. Uniform speed was the first to go, replaced early in his career by the idea that the planets move faster the closer to the sun (the "equal areas in equal times" law). But it took years of struggle to give up the circle and move to the oval and finally the ellipse. As Hanson (1958, p.4) notes, "Before Kepler, circular motion was to the concept of a planet as 'tangibility' is to our concept of 'physical object'".

Kepler also abandoned the idea of an externally perfect universe set in motion once and for all and now running externally in unchanging paths. He abandoned both the idea of the planets' orbits as crystalline spheres containing the planets and that of the orbits as eternal paths traveled by planetary intelligences. Instead he came to see them as paths continually negotiated between the sun and the planets. As Toulmin and Goodfield (1961) noted, "One cannot find before Kepler any clear recognition that the heavenly motions called for an explanation in terms of a *continuously* acting physical force" (p. 201, emphasis in original). Another major conceptual change is Kepler's change from the *anima motrix* - a spirit in the sun that could move the planets - to the *vis motrix* or *virtus motrix* - a physical force or power. This is clearly an ontological change, an instance of what Thagard (1992) calls "branch jumping."

BEFORE	AFTER
Planetary system is governed by mathematical laws	Planetary system is governed by physical causality
Planets' orbits are crystalline spheres containing planets or eternal circles traveled by planetary intelligences	Planets' orbits are paths continually negotiated between the Sun and the planets
Celestial phenomena are separate from earthly physics	Terrestrial knowledge extends to astronomical phenomena
Planetary paths are perfect circles of uniform speed	Planetary paths are ellipses, faster when closer to the Sun and slower when further from the Sun
Anima motrix as "spirit" in Sun that moves planets	*Vis motrix* as "force" from Sun that moves planets

Table 1. Conceptual Change.

Analogical cognition. It seems clear that analogical thinking played a big part in Kepler's discoveries. How do these processes work? The defining characteristic of analogy is that it involves an alignment of relational struc-

ture. There are three psychological constraints on this alignment. First, the alignment must be structurally consistent: i.e., it must observe parallel connectivity and one-to-one correspondence: Parallel connectivity requires that matching relations must have matching arguments; and one-to-one correspondence limits any element in one representation to at most one matching element in the other representation (Gentner and Gentner, 1983). A second characteristic of analogy is relational focus: analogies must involve common relations but need not involve common object descriptions The final characteristic of analogy is systematicity: analogies tend to match connected systems of relations (Gentner, 1983). The systematicity principle captures a tacit preference for coherence and causal predictive power in analogical processing. This implicit preference for aligning connected systems permits the generation of spontaneous *candidate inferences*. By aligning a causal system in the base domain with a (typically less complete) system in the target domain, further statements that are connected to the system in the base can be projected into the target.

There are at least six ways in which the process of analogical comparison can lead to knowledge change (Gentner and Wolff, 2000; Gentner, Brem, Ferguson, Markman, Levidow, Wolff, and Forbus, 1997): (1) highlighting and schema abstraction - extracting common systems from representations, thereby promoting the disembedding of subtle and possibly important commonalities (including common relational systems); (2) projection of candidate inferences from one domain to the other; (3) noticing alignable differences - becoming aware of contrasts on dimensions or predicates that are present in both analogs and/or that are connected to the common structure; (4) re-representation - altering one or both representations so as to improve the match (and thereby, as an important side effect, promoting representational uniformity); (5) incremental analogizing: extending the mapping by returning to the base domain for more material to add to the analogy; and last, the rarest of these, (6) re-structuring - altering the domain structure of one domain in terms of the other 9 though this process probably requires processes besides analogy, as discussed by Nersessian (1992). I focus here on the first four processes, which enable a learner to notice abstract commonalities, to make new inferences, and even to reconceive the concepts in the domain. Let us take these in turn, using the operation of the Structure-mapping engine (SME) as a specific model of the kinds of mechanisms that may be operating (Falkenhainer, Forbus, and Gentner, 1989).

Highlighting commonalities may seem like a rather mundane learning process, but this is not true in the case of common relations. SME's alignment process, taken as a model of human processing, suggests that the act of carrying out a comparison promotes structural alignment and renders the common structure more salient. There is considerable evidence for this claim

(Gentner and Wolff, 2000; Gick and Holyoak, 1983). When a learner is induced to compare two things - whether because of a direct instruction to compare, or because of common labels, or chance juxtaposition - the alignment process renders the common relational structure more salient and prompts a re-representation at a more abstract level (Gentner and Medina, 1998; Loewenstein, Thompson, and Gentner, 1999).

Achieving a structural alignment not only invites abstracting the commonalities but also invites *inferences* from the more structurally complete system to the less-understood system. This kind of inference projection can add new knowledge. Inspired by the analogy with light, Kepler mapped a precursor of the inverse square law from light to the motive power.

Alignable differences are differences connected to commonalities - typically, the same dimension or predicate with two different values and emerge from the alignment process. Interestingly, it is easier to find differences for high-similarity pairs than for low-similarity pairs (Markman and Gentner, 1993). For example, participants given a high-similarity pair like *hotel* and *motel* readily list (alignable) differences: "hotels are in cities, motels are on the highway; you stay longer in hotels than in motels; hotels have many floors, motels only one or two;" and so on. In contrast, when given a low-similarity pair like *magazine/kitten,* subjects tend to list nonalignable differences, such as "you pet a kitten; you don't pet a magazine;" or "kittens have fur and magazines don't;" This is because they are useful in forcing the clarification and delineation of the analogy and in revealing significant differences. For example, Kepler asked whether the motive power could undergo an eclipse. He decided not, and used this difference to conclude that the motive power cannot be the same thing as the sun's light.

Re-representation occurs most typically when two analogs that share considerable structure nonetheless contain a stubbornly different pair of predicates which, if they matched, would allow a much larger march. In this case the learner is motivated to try to recast one or both predicates so as to find an identity. For example, given "the sun propels Jupiter forward" and "the sun pushes Venus around" one might re-represent these relations as "the sun moves (Jupiter/Venus) around". In SME, this is expressed as the principle of *tiered identity.* An analogy must begin with some identical predicates in order to be viable. (Otherwise we could entertain vacuous analogies such as "Fred loves New York" and "The planets revolve around the sun"). given a set of initial identities, people tend to seek additional identities, so that the end result is a larger match. One effect of this implicit drive to increase the match is that it promotes representational uniformity. Two phenomena are likely to be represented more similarly after an analogy than they were before. Finally, analogical change sometimes involves *re-structuring*, in which significant changes of structure occur. This is of course quite rare - if it were

not there would be no stability to mental life. It is doubtful that restructuring occurs in one pass of an analogy. Rather, I suspect it requires repeated working through either the same analogy or, as in Kepler's case, a set of interrelated analogies.

Induction and abduction: As noted above, the analogical processes of highlighting and schema abstraction provide a mechanism by which inductive abstraction can take place. Structure-mapping permits the induction of structured abstractions, not merely sets of common features. This is important for modeling scientific reasoning. Structure-mapping can also capture analogical abduction - that is, it can be used to infer the explanation for a set of observations in a domain (Peirce, 1935; see also Magnani, 2001).

Kepler's analogy between the light and the *anima motrix* led to a massive abduction - the conjecture that the causal relation between the *anima motrix* and the planets's motion was analogous to the causal relation between the sun's light and the illumination of the planets by the sun. Table 2 shows SME's mapping of this analogy, with the abduction in bold (see Gentner et al., 1997, for details).

LIGHT	ANIMA MOTRIX
Planet	Planet
Sun	Sun
Space	Space
Distance (planet, sun)	*Distance* (planet, sun)
Brightness (planet)	*Speed* (planet)
PROMOTE (*Brightness* (planet))	**PROMOTE (*Speed* (planet))**
PRODUCE (sun, light)	**PRODUCE (sun, anima)**
Concentration	**Concentration**
(light, planet)	**(anima, planet)**
Distance *(planet, sun)*	***Distance* (planet, sun)**
TRAVEL	**TRAVEL**
(light, sun, planet, space)	**(anima, sun, planet, space)**
REACH (light, planet)	**REACH (anima, planet)**
CAUSE	**CAUSE**
(TRAVEL	**(TRAVEL**
(light, sun, planet, space),	**(anima, sun, planet, space)),**
REACH (light, planet),	**REACH (anima, planet))**
CAUSE	**CAUSE**
(REACH (light, planet),	**(REACH (anima, planet),**
PROMOTE (*Brightness* (planet))	**PROMOTE (*Speed* (planet))**
QROP-	QROP-
(*Brightness* (planet),	(*Speed* (planet),
Distance (planet, sun))	*Distance* (planet, sun))

Table 2. Simulating Kepler's analogy: the mapping produced by SME. N.B: Bold text marks the abduction. *QPROP* means "qualitatively proportional to" (Forbus, 1984).

Kepler used a large number of analogies for the sun's power, returning again and again to three main analogies. The first viewed the sun's power as analogous to light, as in Table 2 - a power that propelled the planets around in their orbits. A second analogy viewed the sun's power as magnetism - a new phenomenon just being explored by Gilbert in *De magnete* (1600). In this analogy, Kepler likened the sun and planet to two magnets that approach or repel each other depending on which poles are proximate. The third analogy viewed the sun's power as a kind of current in which the planets navigated as boatmen. This last was used to explain why the planets move closer and further on their orbits instead of maintaining a constant distance from the sun. He postulated that as the sun rotates around its axis, it creates a whirling circular river of motive power that pushes the planets around. (In Kepler's pre-Newtonian physics, the sun was required to push the planets around in their orbits, not merely to attract them.) Then, as a boatman can steer his boat sidewise to the river's current, so the planets could move in and out across this current of motive power. But although Kepler worked with this analogy for decades, he was never satisfied with it; it seemed to require an excessive degree of sentience on the part of the planets to sense how to steer. He articulated this challenge in the *Astronomia nova* (1609; quoted in Koestler, 1978, p. 342):

> Kepler, does't thou wish then to equip each planet with two eyes? Not at all. For it is not necessary, either, to attribute them feet or wings to enable them to move [...]. Our speculations have not yet exhausted all Nature's treasures, to enable us to know, how man senses exist [...]. We are discussing natural matters of much lower rank: forces which do not exercise free will when they change their activities, intelligences which are by no means separate from, but detached to, the stellar bodies to be moved, and are one with them.

Was Kepler's use of analogy apparent or real? I have suggested that analogical processes of highlighting, projection, re-representation and drawing alignable differences operate to bring about conceptual change in Kepler's work. But were the analogies in Kepler's works and journals integral to his thought? Might they not have been designed after the fact, for purely communicative or persuasive purposes? There are reasons to think Kepler's analogies were just what they seem - tools central to his thought. First, Kepler was an open, inclusive writer, whose writings are unusually rich in descriptions of his thought processes. Many of Kepler's commentators have noted the exceptional - at times even excessive - candor and detail of his scientific writing. Holton, 1973, p.69-70 notes that

> [Modern scientists are] [...] taught to hide behind a rigorous structure the actual steps of discovery - those guesses, errors, and occasional strokes of

good luck without which creative scientific work does not usually occur. But Kepler's embarrassing candor and intense emotional involvement force him to give us a detailed account of his tortuous process [...]. He gives us lengthy accounts of his failures, though sometimes they are tinged with ill-concealed pride in the difficulty of his task.

Kepler was explicit in his intention to share the difficulties of discovery: "I therefore display these occasions [errors and meanderings] scrupulously, with, no doubt, some attendant difficulty for the reader. Nevertheless, that victory is sweeter that was born in danger, and the sun emerges from the clouds with redoubled splendour" (*Astronomia nova*, 1609, p. 95). He frequently included long, tedious sections of calculations that turned out to lead nowhere, informing the reader only afterward that the reasoning had been wrong from the start. In the midst of one such section he wrote, "If this wearisome method has filled you with loathing, it should more properly fill you with compassion for me, as I have gone through it as least seventy times" (p. 256). This is not to say that Kepler's writings are pure diaries; some filtering and organizing surely took place. But his fascination with the process of discovery led him to preserve much of the trail. Indeed, Kepler had the rather touchingly optimistic belief that readers would wish to follow "the roads by which men arrive at their insights into celestial matters." In the introduction to the *Astronomia nova* (Kepler, 1609/1992) he states this agenda - characteristically, in an extended analogy:

Here it is a question not only of leading the reader to an understanding of the subject matter in the easiest way, but also, chiefly, of the arguments, meanderings, or even chance occurrences by which I the author first came upon that understanding. Thus, in telling of Christopher Columbus, Magellan, and of the Portuguese, we do not simply ignore the errors by which the first opened up America, the second, the China Sea, and the last, the coast of Africa; rather, we would not wish them omitted, which would indeed be to deprive ourselves of an enormous pleasure in reading (p. 78).

Kepler's openness extended beyond sharing the process of discovery. He also provided a running account of his feelings about the work, including the kind of emotional remarks that no modern scientist would consider publishing. The open spontaneity of Kepler's style suggests that his writings were in good measure directly reflective of his thinking. For example,

If I had embarked upon this path a little more thoughtfully, I might have immediately arrived at the truth of the matter. But since I was blind from desire [to explain the deviation from a circular orbit] I did not pay attention to each and every part [...] and thus entered into new labyrinths, from which we will have to extract ourselves. (Kepler, 1609/1992, pp. 455-456) or from the same work,

Consider, thoughtful reader, and you will transfixed by the force of the argument [...]. And we, good reader, will not indulge in this splendid triumph for more than one small day [...] restrained as we are by the rumours of a new rebellion, lest the fabric of our achievement perish with excessive rejoicing (p. 290).

Beyond the general openness, there are several specific signs that Kepler used analogies to think with and not simply to explain. First, Kepler pursued his major analogies with intensity within and across his major works - spelling out commonalities and differences, inferences, and incremental extensions. Not only did he push the analogical mappings, he also sought further information about both sides of the analogy. For both his major analogies, he delved energetically into the base domain. For the magnetism analogy, he studied Gilbert's *De magnete*. For the light analogy, he investigated optics and wrote two treatises (the *Astronomiae pars optica* in 1604 and the *Dioptrice* in 1610). He used this information both to challenge and to extend the analogy. For example, he devoted many pages to exploring whether the magnetism-*vis motrix* comparison was purely an analogy or might instead be the causal means by which the Sun influenced the planets.

Further, Kepler made several direct statements that he used analogy. For example, he justified his analogy between light through a lens and the geometry of conic sections and as follows:

But for us the terms in Geometry should serve the analogy (for I especially love analogies, my most faithful masters, acquainted with all the secrets of nature) and one should make great use of them in geometry, where - despite the incongruous terminology - they bring the solution of an infinity of cases lying between the extreme and the mean, and where they clearly present to our eyes the whole essence of the question (Kepler, *The Optics*, 1904; quoted in Vickers, 1984, pp. 149-150).

Another indication that Kepler actually used analogy in reasoning is that he wrote explicitly on how best to use analogy in scientific thinking. In fact, the improper use of analogy was one of the few areas in which Kepler - normally generous towards other scientists - showed a sharply critical side. For example, he criticized the use of symbols as informative analogies by hermeticists and alchemists (for instance, the claim that iron must be corrosive since its (al)chemical symbol has a sharp point): "I have shown that Ptolemy luxuriates in using comparisons in a poetical or rhetorical way, since the things that he compares are not real things in the heavens". He also wrote: "I too play with symbols, and have planned a little work, *Geometric Cabala*, which is about the Ideas of natural things in geometry; but I play in such a way that I do not forget that I am playing. For nothing is proved by symbols [...] unless by sure reasons it can be demonstrated that they are not

merely symbolic but are descriptions of the ways in which the two things are connected and of the causes of this connexion." This is essentially the modern conception of analogical reasoning.

A further indication that analogy was a natural mode of thought for Kepler is that he used analogy prolifically. Kepler drew on a variety of base domains in his work - a boat in a current, magnets, a balance beam, an orator, and so forth. He also used them in informal writing, in personal letters and journals. Complaining of having to make astrological forecasts to please his patrons he wrote (Kepler, 1606, *De Stella nova in pede Serpentarii)*:

> A mind accustomed to mathematical deduction, when confronted with the faulty foundations [of astrology] resists a long, long time, like an obstinate mule, until compelled by beating and curses to put its foot into that dirty puddle.

He often engaged in analogical play, as for example in the preface to the *Astronomia nova* (1620; 1992 pp. 30-33). Describing his solution to the long-unsolved problem of the orbit of Mars, he analogized Mars to a captive of war:

> I am now at last exhibiting for the view of the public a most Noble Captive, who has been taken for a long time now through a difficult and strenuous war waged by me under the auspices of Your Majesty [...]. It is he who is the most potent conqueror of human inventions, who, ridiculing all the sallies of the Astronomers, escaping their devices, and striking down the hostile throngs, kept safe the secret of his empire, well guarded throughout all ages past, and performed his rounds in perfect freedom with no restraints [...]. In this place chief praise is to be given to the diligence of Tycho Brahe, the commander-in-chief in this war, who [...] explored the habits of this enemy of ours nearly every night for twenty years, observed every aspect of the campaign, detected every stratagem, and left them fully described in books [...] I [...] directed the Brahean machines thither, equipped with precise sights, as if aiming at a particular target, and besieged each position with my enquiry [...]. This campaign did not, however, succeed without sweat [...] [because of] the enemy's enterprise in making sallies, and his vigilance for ambuscades [...] whenever he was driven or fled from one castle, he repaired to another, all of which required different means to be conquered [...]. At last, when he saw that I held fast to my goal, while there was no place in the circuit of his kingdom where he was safe or secure, the enemy turned his attention to plans for peace [...] and, having bargained for liberty within limits subject to negotiation, he shortly thereafter moved over most agreeably into my camp with Arithmetic and Geometry pressing closely by his sides.

Kepler contrasted with modern scientists. As noted above, Kepler's way of using analogy contrasted sharply with that of the alchemists who preceded him. How does it compare with the approach of modern scientists? Empirical studies of learning and problem - solving have shown that analogies to prior knowledge can foster insight into new material (Bassok and Holyoak, 1989; Gentner and Gentner, 1983; Gick and Holyoak, 1983). These laboratory results are supported by observations of the scientific process (Dunbar, 1995; Nersessian, 1984, 1986; Thagard, 1992).

Dunbar's (1994, 2001) detailed observations of the online process of discovery in four different microbiology laboratories have led him to a characterization of what makes for a creative laboratory. He suggests that the highly productive microbiology laboratories are those that use analogies in quantity and take them seriously. In the successful lab groups, analogies are extended and "pushed" in group discussions. This is the most direct evidence to date that the process of working through an analogy contributes to scientists' on-line creative thinking, and it lends force to Kepler's introspection that analogy furthered - and perhaps even engendered - his theories.

Attention to inconsistencies is another common factor. Dunbar noted that creative laboratories seize on inconsistencies between results and predictions and between observations that should have agreed. and work on explaining them. Kepler shows the same aesthetic. A well-known instance is his rejection of his own early model of Mars's orbit after several years of work due to a discrepancy of only 8 minutes of arc with Tycho Brahe's observations – which led him with great trepidation to abandon the idea of circular orbits. But there were many others. His initial reformulation of Copernicus's model was driven in part by its inconsistent treatment of Mercury's orbit and its continued use of epicycles. In both cases, I suggest that attention to inconsistencies acts as a *motivator* for conceptual change, whereas analogy acts as a *process* by which conceptual change occurs.

Dunbar's third claim is that creative labs have interaction patterns that bring together heterogeneous knowledge, partly because it increases the range of different analogies that are used. Here too historical period is a factor. There was no clear division of disciplines in 1600. A Kepler or a Galileo could command enough broad knowledge to create his heterogeneous pool of analogies. Kepler's published works, besides his great works on celestial physics, included papers on optics, the nature of comets, the birthdate of Jesus, and a new method of measuring wine casks (in which he developed a method of infinitesimals that took a step towards the calculus). Such diversity within an individual might be analogous to the heterogeneity of background Dunbar noted in his successful lab groups.

There are also some interesting differences between Kepler and Dunbar's microbiologists. By far the vast majority of the analogies Dunbar observed

were close literal similarities (what he calls *local analogies*), typically involving the same kind of organism or species, similar diseases or genetic materials, etc. Kepler also used close analogs on many occasions. When he first noticed the regularity that speed diminishes as distance from the sun increases, he immediately asked whether this between-planet pattern would apply within planets. This led to the inference that each planet moves fastest when it is closest to the sun, and this in turn led to the first statement of the equal-areas in-equal-times law. By this analogy, early in his career he was able to give up the assumption of uniform speed. Another close analogy is a kind of reverse analogy he used in the calculation of Mars's orbit - he imagined how the earth's orbit would appear from Mars. A third instance is a mapping from the sun's power affecting the planets to infer that the earth's pwer would affect the moon. This led him to suggest that the moon causes the earth's tides.

But despite these similarities, Kepler's analogizing is strikingly different from that of the microbiologists in one important way. Unlike Dunbar's microbiologists, who use distant analogies only for local illustrative points, Kepler used many distant analogies as integral to his work. We could attribute this difference to the contrast between a rare genius and a group of merely excellent scientists. But it could also stem from a difference in the evolutionary stage of science. Kepler's self-appointed task was to invent the science of astrophysics - or as he put it: "Ye physicists, prick your ears, for we are about to invade your territory". The task was all the more challenging in that there was no well-established physics. Given this underdeveloped state of affairs, distant analogies were in many cases his only option. There was no literal similarity to be had. In contrast, in the microbiology laboratories that Dunbar studies, the historical stage is one of a fairly well developed (but not yet fully explored) framework. At this stage, there are many close analogs that are likely to be fruitful. In sum, I suggest that close analogies and far analogies may be characteristic of within-paradigm and between-paradigm stages in the history of a field, respectively (Kuhn, 1962). Local analogies fill in an established framework; distant analogies create new frameworks.

Creativity, fluidity and structure. Creativity is often characterized as a kind of conceptual fluidity in which concepts flow together and boundaries are constantly shifting. But if we take the notion of "fluidity" seriously, it seems exactly the wrong idea. I suggest that a better model of the creative process begins with a representational structure and alters that structure, sometimes locally and sometimes radically. To bear down on the fluid analogy for a moment, moving a bit of liquid from one part of a large body to another yields no discernible difference. In contrast, moving or replacing part of a structure leads to a noticeably different structure. In other words,

structured belief representations before and after the change are what makes the change matter.

Kepler's conceptual changes proceeded from alignment and mapping between highly structured representations. His discoveries are best modeled by a structure-sensitive process. For example, SME, using structured representations, behaves in what could be considered a creative manner when it derives cross-dimensional structural matches, projects candidate inferences, infers new entities and incrementally extends its mapping. Further, this model predicts that minor alternations - like noting the abstraction or mapping an inference between two close analogs - should be frequent and easy, whereas radical restructuring is rare but possible. I suggest that structured representations and structure-sensitive alignment and mapping processes are instrumental in conceptual change and discovery. Analogy is an engine of creativity in part because it preserves a fair degree of structure while inviting alterations.

One more aspect of analogical creativity needs comment. I have focused here on the use of analogies in online thought - that is, the processes of analogical reasoning once one has both analogs in mind. But it is obviously crucial to ask how potential analogs come to mind. The processes that govern access and reminding to prior potential analogs are far less under the control of the reasoner than those that govern online mapping. To use an analogy I've used before, gaining access to long-term memory remindings is like baiting a hook and throwing it into a vast ocean. There is no telling which fish, if any, will bite. Indeed, remindings can occur any time, even when the person is otherwise occupied. However, the example of Kepler suggests that there are practices that foster analogical remindings. The first point is rather obvious: thinking intensely about a domain increases the probability of remindings to that domain. (I experienced a striking (but non-productive) instance of this phenomenon the day of the World Trade enter attack when, seeking distraction, I weeded the garden and experienced roughly 15 remindings from pulling weeds to terrorism). The second point is that pushing hard on the analogies that do occur increases both the likelihood and the quality of further remindings. The more one articulates and abstracts the relational structure of a domain, the more it can call forth remindings to other relationally similar knowledge. I speculate that creativity involves a kind of iterative process of pondering, retrieval, intensive mapping, and re-retrieval. From this point of view, Kepler's genius owes as much to passionate labor as to creative leaps.

REFERENCES

Barker, P., 1991, Stoic contributions to early modern science, in: *Atoms, Pneuma, and Tranquility: Epicurean and Stoic Themes in European Thought*, M.J. Osler, ed., Cambridge University Press, Cambridge, MA, pp. 135-154.

Bassok, M. and Holyoak, K.J., 1989, Interdomain transfer between isomorphic topics in algebra and physics, *Journal of Experimental Psychology: Learning, Memory, and Cognition* 15:153-166.

Dunbar, K., 1995, How scientists really reason: Scientific reasoning in real-world laboratories, in: *The Nature of Insight*, R.J. Sternberg and J.E. Davidson, eds., MIT Press, Cambridge, MA, pp. 365-395.

Falkenhainer, B., Forbus, K.D., and Gentner, D., 1989, The structure-mapping engine: Algorithm and examples, *Artificial Intelligence* 41:1-63.

Forbus, K.D., 1984, Qualitative process theory, *Journal of Artificial Intelligence* 24: 85-168.

Gentner, D., 1982, Are scientific analogies metaphors, in: *Metaphor: Problems and Perspectives*, D.S. Miall, ed., Harvester Press, Brighton, England, pp. 106-132.

Gentner, D., 1983, Structure-mapping: A theoretical framework for analogy, *Cognitive Science* 7:155-170.

Gentner, D., Brem, S., Ferguson, R., Wolff, P., Levidow, B.B., Markman, A.B., and Forbus, K., 1997, Reasoning and conceptual change: A case study of Johannes Kepler, *Journal of the Learning Sciences* 6(1): 3-40.

Gentner, D., Brem, S., Ferguson, R.W., Wolff, P., Markman, A.B., and Forbus, K.D., 1997, Analogy and creativity in the works of Johannes Kepler, in: *Creative Thought: An Investigation of Conceptual Structures and Processes*, T. B. Ward, S. M. Smith, and J. Vaid, eds., American Psychological Association, Washington, DC, pp. 403-460.

Gentner, D., and Gentner, D.R., 1983, Flowing waters or teeming crowds: Mental models of Electricity, in *Mental Models*, D. Gentner and A. L. Stevens, eds., Erlbaum, Hillsdale, NJ, pp. 99-129.

Gentner, D., and Jeziorski, M., 1993, The shift from metaphor to analogy in western science, in *Metaphor and Thought*, A. Ortony, ed., Cambridge, England, pp. 447-480.

Gentner, D., and Medina, J., 1998, Similarity and the development of rules, *Cognition* 65: 263-297.

Gentner, D., and Wolff, P., 2000. Metaphor and knowledge change, in: *Cognitive Dynamics: Conceptual Change in Humans and Machines*, E. Dietrich and A. Markman, eds., Lawrence Erlbaum Associates, Mahwah, NJ, pp. 295-342.

Gick, M.L., and Holyoak, K.J., 1983, Schema induction and analogical transfer, *Cognitive Psychology* 15: 1-38.

Gingerich, O., 1993, *The Eye of Heaven*, The American Institute of Physics, New York.

Hanson, N.R., 1958, *Patterns of Discovery: An Inquiry into the Conceptual Foundations of Science*, Cambridge University Press, Cambridge.

Holton, G., 1973, *Thematic Origins of Scientific Thought*, Harvard University Press, Cambridge. MA.

Holyoak, K.J., and Thagard, P.R., 1995, *Mental Leaps: Analogy in Creative Thought*, MIT Press, Cambridge, MA.

Kepler, J., 1969, *Epitome of Copernican Astronomy*, (tranlated by C.G. Wallis), Kraus Reprint Co., New York.

Kepler, J., 1981, *Mysterium Cosmographicum*, (translated by A.M. Duncan, 2nd ed.), Abaris Books, New York, (original work published 1596, annotated 1621).

Kepler, J., 1992, *New Astronomy* (translated by W. Donahue), Cambridge University Press, Cambridge, (original work published 1609).

Koestler, J.L., 1968, *The Sleepwalkers*, Grosset and Dunlap, The Universal Library, New York.

Kuhn, T.S., 1962, *The Structure of Scientific Revolutions*, University of Chicago Press, Chicago.

Loewenstein, J., Thompson, L., and Gentner, D., 1999, Analogical encoding facilitates knowledge transfer in negotiation, *Psychonomic Bulletin and Review* 6(4): 586-597.

Magnani, L., 2001, *Philosophy and Geometry: Theoretical and Historical Issues*, Kluwer Academic Publishers, Dordrecht.

Markman, A.B., 1997, Constraints on analogical inference, *Cognitive Science* 21(4): 373-418.

Markman, A.B., and Gentner, D., 1993, Splitting the differences: A structural alignment view of similarity, *Journal of Memory and Language* 32: 517-535.

Nersessian, N.J., 1984, *Faraday to Einstein: Constructing Meaning in Scientific Theories*, Nijhoff, Dordrecht.

Nersessian, N.J., 1986, A cognitive-historical approach to meaning in scientific theories, in: *The Process of Science: Contemporary Philosophical Approaches to Understanding Scientific Practice*, N. J. Nersessian, ed., Nijhoff, Dordrecht.

Nersessian, N.J., 1992, How do scientists think? Capturing the dynamics of conceptual change in science, in: *Cognitive Models of Science*, R.N. Giere and H. Feigl, eds., University of Minnesota Press, Minneapolis, pp. 3-44.

Peirce, C.S., 1934-35, *Collected Papers*, Harvard University Press, Cambridge, MA.

Thagard, P., 1992, *Conceptual Revolutions*, Princeton University Press, Princeton, NJ.

Toulmin, S. and Goodfield, J., 1961, *The Fabric of the Heavens*, Harper, New York.

Vickers, B., 1984, Analogy versus identity: The rejection of occult symbolism, 1580-1680, in: *Occult and Scientific Mentalities in the Renaissance*, B. Vickers, ed., Cambridge University Press, Cambridge, pp. 95-163.