Carey's book is an outstanding contribution to cognitive development (Carey 2009). It reviews and updates findings that infants and young children have abstract or "core" representations of objects, agents, number, and causes. The number chapters feature the argument for discontinuity between infant and later cognitive development. They include evidence that infants use two separate number abstraction systems: an object-file, parallel system for the small numbers of 1 to 3 or 4; and a ratio-dependent quantity mechanism for larger numbers. This contrasts with adults, who use a ratio-dependent mechanism for all values (Cordes et al. 2001).

Further, Carey argues that verbal counting is first memorized without understanding and that the meaning of counting and cardinality is embedded in the learning of the quantifier system. She cites Wynn's (1990; 1992) "Give X" and LeCorre and Carey's (2007) tasks that children aged 2 years, 8 months to 3 years, 2 months typically fail as well as analyses on quantifiers, including *some* and *many*.

An alternative account runs as follows. Infants possess a core domain for arithmetic reasoning about discrete and continuous quantity, necessarily including both mechanisms for establishing reference and mechanisms for arithmetic reasoning. The nonverbal domain outlines those verbal data and uses rules that are relevant to its growth. The development of adult numerical competence is a continuous and sustained learning involving the mapping of the cultural system for talking about quantity into the inherited nonverbal system for reasoning about quantity. Counting principles constitute one way to establish reference for discrete quantity because they are consistent with and subservient to the operations of addition and ordering, that is, they are consistent with basic elements of arithmetic reasoning. In this view, the Carey account focuses too much on reference and almost ignores the requirement that symbols also enter into arithmetic reasoning. The well-established ability of infants and toddlers to recognize the ordering of sequentially presented numerosities, including small ones, requires a counting-like mechanism to establish reference. If the symbols that refer to numerosities do not enter into at least some of the operations that define arithmetic (order, addition, subtraction), then they are not numerical symbols. However, there is evidence that beginning speakers recognize that counting yields estimates of cardinality about which they reason arithmetically.

1. Infants can represent numerosity in the small number range. Cordes and Brannon (2009) show that, if anything, numerosity is more salient than various continuous properties in the 1–4 number range. Converging evidence is found in VanMarle and Wynn (in press).

2. Cordes and Brannon (2009) also show that 7-month-old infants discriminate between 4:1 changes when the values cross from small (2) to larger (8) sets. These authors conclude that infants can use both number and object files in the small *N* range, a challenge to the view that there is a discontinuity between the small number and larger number range for infants.

3. Two-and-a-half-year-olds distinguish between the meaning of "a" and "one" when tested with the "What's on the card WOC?" task (Gelman 1993). When they reply to the WOC question with one item, they often say "a __". When told "that's a one-x card", the vast majority of $2\frac{1}{2}$-year-olds both counted and provided the cardinal value on set sizes 2 and 3 and young 3-year-olds ($\leq$3 years, 2 months) provide both the relevant cardinal and counting solution for small sets as well as some larger ones. Syrett et al. (in press) report comparable or better success rates for children in the same age ranges. The appearance of counting when cardinality is in question is good evidence that these very young children, who can be inconsistent counters, nonetheless understand that counting renders a cardinal value.

4. Arithmetic abilities appear alongside early counting. Two-and-a-half-year-old children transferred an ordering relation between 1 versus 2 to 3 versus 4 (Bullock & Gelman 1977). When these children encountered the unexpected change in numerosities, they started to use count words in a systematic way. This too reveals an understanding of the function of counting well before they can do the give-*N* task. Carey's claim that "originally the counting routine and the numeral list have no numerical meaning" (p. 311) is simply false.

5. Gelman's magic show was run in a number of different conditions and with 3-year-old children. Children this age distinguished between operations that change cardinal values (numerosities) and those that do not, across a number of studies. Moreover, when the cardinality of the winner comes into question, they very often try to count the sets, which are in the range of 2–4, and occasionally 5.

6. Further evidence that 3-year-olds understanding of cardinality comes from the Zur and Gelman (2006) arithmetic–counting task. Children started a round of successive trials with a given number of objects, perhaps doughnuts, to put in their bakery shop. They then sold and acquired 1–3 doughnuts. Their task was to first predict – without looking – how many they would have, and then to check. Their predictions were in the right direction, if not precise. They counted to check their prediction and get ready for the next round. They never mixed the prediction–estimation phase and the checking phase. Counts were extremely accurate and there was no tendency to make the count equal the prediction. Totals could go as high as 5.

7. The idea that understanding of the exact meaning of cardinal terms is rooted in the semantics of quantifiers is challenged in Hurewitz et al. (2006). They found that children in the relevant age range were better able to respond to exact number requests (2 vs. 4) than to "some" and "all."

8. An expanded examination of the Childes database with experiments with the partitive frame (e.g., *zav of Y*) and modification by the adverb *very* (e.g., *very zav*) reveal that the Bloom and Wynn analysis of semantics is neither necessary nor sufficient to accomplish the learnability challenge (Syrett et al., in press).

The preverbal arithmetic structure can direct attention to and assimilate structurally relevant verbal data and their environments.

Language and analogy in conceptual change

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Dedre Gentner and Nina Simms
Department of Psychology, Northwestern University, Evanston, IL 60208.
gentner@northwestern.edu
http://groups.psych.northwestern.edu/gentner/
ninasimms@northwestern.edu

Abstract: Carey proposes that the acquisition of the natural numbers relies on the interaction between language and analogical processes: specifically, on an analogical mapping from ordinal linguistic structure to ordinal conceptual structure. We suggest that this analogy in fact requires several steps. Further, we propose that additional analogical processes enter into the acquisition of number.

How humans come to possess such striking cognitive abilities and rich conceptual repertoires is perhaps *the* question of cognitive science. Susan Carey has explored this question in part by identifying specific areas – such as number – in which humans demonstrate unique and impressive ability, and exploring their development in great depth (Carey 2009). In her treatment of number, Carey argues that children gain an understanding of the natural numbers through a process of mapping the ordinal structure of the number list to quantity. We agree with this proposal, but we suggest (1) that analogy interacts with language in several additional and distinct ways to support the acquisition of number; and (2) that arriving at the analogy from ordered numerals to ordered quantities probably requires more than a single leap (Gentner 2010).

Carey and others have provided substantial evidence suggesting that two core capacities enter into numerical cognition: the analog magnitude system, which allows approximate judgments of quantity; and a system for keeping track of small numbers of items (up to three or four). Neither is sufficient for representing large exact numbers—nor indeed for representing the natural number sequence at all. As Carey reviews, two lines of evidence suggest that language is key in this achievement: (1) cross-linguistic studies of Amazonian peoples whose languages – Pirahã and Mundurukú – lack a full counting system, and who show marked deficiencies in dealing with exact numerosities greater than 3 (Everett 2005; Frank et al. 2008; Gordon 2004; Pica et al. 2004); and (2) developmental evidence that children at first learn the linguistic count sequence as a kind of social routine (Fuson 1988), and that learning this sequence is instrumental in their acquisition of the conceptual structure of the natural numbers.

We believe language is instrumental in achieving conceptual mastery of all sorts; indeed, we have proposed that language, combined with powerful analogical processes, is crucial to humans' remarkable abilities (Gentner & Christie 2008). Analogical processes conspire with language in several ways in conceptual development, including the acquisition of number. First, common labels invite comparison and subsequent abstraction (Gentner & Medina 1998). Hearing the count label "3" applied to three pears and three apples prompts comparison across the sets and abstraction of their common set size (Mix et al. 2005). Second, the repeated use of the same numerals for the same quantities helps stabilize numerical representation. English speakers consistently assign "3" to the same quantity, whether counting up from 1 or down from 10. This uniform usage might be taken for granted, except that Pirahã speakers, astoundingly, assign their numeral terms to different quantities when naming increasing vs. decreasing set sizes. A third way in which analogy interacts with language to support cognitive development is that linguistic structure invites corresponding conceptual structure (Gentner 2003). For example, learning and using the spatial ordinal series "top, middle, bottom" invites preschoolers to represent space in an ordered vertical pattern (Loewenstein & Gentner 2005).

This brings us to Carey's (2004; 2009) bold proposal that learning the natural numbers relies on an analogical mapping from ordinal linguistic structure to ordinal conceptual structure. One symptom of this analogical insight is a sudden change in the pace of learning. As Carey reviews, children first learn the count sequence as a social routine. Despite their fluency with this linguistic sequence, children may show only minimal insight into the binding to numerical quantity. A typical 2-year-old can count from "1" to "10," but cannot produce a set of five items on request. The binding of small numerals to quantities is slow, piecemeal, and context-specific (Mix et al. 2005; Wynn 1990). But once a child binds "3" or "4" to the appropriate quantity, the pattern changes; the child rapidly binds the succeeding numbers to their cardinalities. Further, the child shows understanding of the successor principle, that every natural number has a successor whose cardinality is greater by one.

How does the analogy between numeral order and quantity order emerge? The correspondence between *counting one further* in the linguistic sequence and *increasing by one* in set size is highly abstract. We suggest that children arrive at this insight in a stepwise fashion, roughly as follows (for simplicity, we consider the case where the insight occurs after "3" is bound to 3):

When "1", "2" and "3" are bound to their respective quantities, the child has two instances in which *further-by-one* in count goes with *greater-by-one* in set size: $1 \rightarrow 2$ and $2 \rightarrow 3$. Should the alert child wonder whether this parallel continues to hold, s/he will find immediate confirmation: counting from "3" to "4" indeed goes with a set size increase $3 \rightarrow 4$. At this point the child has a very productive rule of thumb:

$$\text{IMPLIES}\{\text{FURTHER-BY-ONE}(\text{count list}),\ \text{GREATER-BY-ONE}(\text{set size})\}$$

Over repeated use of this highly productive rule, the child re-represents the two parallel relations as the same (more abstract) relation – a successor relation – applying to different dimensions, such as:

$$\text{GREATER-THAN}[(\text{count}(n), \text{count}(n+1)] \longleftrightarrow \text{GREATER-THAN}[(\text{setsize}(n), \text{setsize}(n+1)]$$

At this point the analogy has revealed a powerful abstraction: the common relational structure required for the successor function.

Bertrand Russell (1920) memorably stated: "It must have required many ages to discover that a brace of pheasants and a couple of days were both instances of the number 2: the degree of abstraction involved is far from easy." Though English speakers may see the natural numbers as obvious, the evidence from the Pirahã bears out Russell's speculation: a conception of "two-ness" is not inevitable in human cognition. Carey's proposal provides a route by which this insight can be acquired.

A unified account of abstract structure and conceptual change: Probabilistic models and early learning mechanisms

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Alison Gopnik
Department of Psychology, University of California at Berkeley, Berkeley, CA 94720.
gopnik@berkeley.edu www.alisongopnik.com

Abstract: We need not propose, as Carey does, a radical discontinuity between core cognition, which is responsible for abstract structure, and language and "Quinian bootstrapping," which are responsible for learning and conceptual change. From a probabilistic models view, conceptual structure and learning reflect the same principles, and they are both in place from the beginning.

There is a deep theoretical tension at the heart of cognitive science. Human beings have abstract, hierarchical, structured, and accurate representations of the world: representations that allow them to make wide-ranging and correct predictions. They also learn those representations. They derive them from concrete, particular, and probabilistic combinations of experiences. But how can we learn abstract structure from the flutter and buzz at our retinas and eardrums? Nativists, from Plato to "core cognition" theorists, argue that it only *seems* that we learn; in fact, the abstract structure is innate. Empiricists, from Aristotle to connectionists, argue that it only *seems* that we have abstract structure; in fact, we just accumulate specific sensory associations. When we see both abstract structure and learning – notably in scientific theory change – traditional nativists and empiricists both reply that such conceptual change requires elaborate social institutions and explicit external representations.

Carey has made major contributions to the enormous empirical progress of cognitive development (Carey 2009). But those very empirical discoveries have actually made the conceptual problem worse. Piaget could believe that children started out with specific sensorimotor schemes and then transformed those schemes into the adult's abstract representations. But Carey's own studies, along with those of others, have shown that this is not a feasible option. On the one hand, contra the empiricists, even infants have abstract structured knowledge. On the other hand, contra the nativists, conceptual theory change based on