

Using Spatial Analogy to Facilitate Graph Learning

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Abstract. Graphical depictions of complex interactions pose a challenge to spatial reasoning. In this research, we tested whether analogical processes can be harnessed to help students learn to solve complex graphical reasoning problems. Specifically, we asked whether a brief training experience using spatial analogies could help students learn about *stock-and-flow* graphs. The basic idea of our intervention was to juxtapose contrastive graphs and encourage students to compare them. In two studies, we test the following predictions derived from structural alignment theories of analogy: (1) comparing contrastive graphs during training will lead to better performance in a graph-understanding task than will studying the same exemplars sequentially; and (2) comparing high-similarity pairs will lead to better performance than will comparing low similarity pairs. The results support both of these predictions, indicating that even a brief analogical comparison task can confer relational insight. Further, these results corroborate prior evidence that a structural alignment process underlies analogical comparison.

Keywords: Analogy, Analogical Comparison, Structural Alignment, Spatial Learning, Graphical Reasoning.

1 Introduction

Comparison of exemplars is a powerful learning process that has been shown to improve learning in a variety of domains. Indeed, according to Gentner [1], “The simple, ubiquitous act of comparing two things is often highly informative to human learners... Comparison is a general learning process that can promote deep relational learning and the development of theory-level explanations” (pp. 247, 251). Analogical comparison has been shown to aid learning across a broad range of topics, ranging from preschoolers learning new words [2] through elementary school children learning estimation methods [3] to business school students learning contract negotiation skills [4]. Within the spatial domain, there is evidence that spatial analogies can help learners to extract and use common spatial structure between two exemplars. For example, preschoolers who are given a challenging mapping task from one model room to another perform better if they first compare two models than if they interact with the same two models one-at-a-time [5].

In this research we asked whether a brief analogical training experience, in which students were encouraged to make comparisons and identify contrasts, could help them learn important relational principles involved in complex graph integration problems. The outline of this paper is as follows. First, we lay out a theoretical framework for this work. We use the structure-mapping theory of analogy, which proposes that analogical comparison involves a process of structural alignment [6-7]. We then review research that illustrates how structural alignment is helpful for learning. Next we propose graph learning as a particularly fruitful domain in which to explore structural alignment as a learning tool, and introduce the specific kind of graphs that we investigated. We then present our experiments and review the results. We consider theoretical and applied implications of our findings, and close with a discussion of study limitations and future directions.

1.1 Analogical Comparison Fosters Learning

Comparison is powerful learning process [3], [7-8]. According to Structure-Mapping Theory (SMT) [6], [9-10], this is because comparison entails a structural alignment process that promotes a focus on common relational structure. This allows learners to move beyond superficial, possibly idiosyncratic features of particular examples [2], [11-12].

Under Structure-Mapping Theory [6], [9-10], carrying out a comparison involves aligning two structured representations so that matching objects and relations are placed into correspondence with one another (structural alignment). Once aligned, inferences can then be projected from one representation to another¹. A key point of SMT is that common relations are more likely to be highlighted during comparison than are common object properties. This is because the structural alignment process favors matches that are connected to other matching information. For example, adults asked to match elements between two pictures are more likely to choose correspondences based on common relational role (rather than matching similar objects) if they have previously compared the two pictures [13].

Structural alignment paves the way for at least three distinct kinds of learning. First, as noted above, structural alignment highlights shared relational structure [4], [8], [13]. This can give rise to a new relational abstraction, which can then be transferred and applied to new situations [4], [8]. Second, rather paradoxically, highlighting commonalities also facilitates noticing differences that are connected to the shared structure, known as alignable differences [14-17]. A third consequence of structural alignment is that inferences may be brought from one situation to the other.

1.2 Analogical Comparison in Spatial Learning

While analogy (structural alignment) is a domain-general process, *spatial analogy* is a fundamental and pervasive kind of analogy. In spatial analogy, one or both analogs contain spatial relations. For example, one can use a cross-domain spatial comparison

¹ Many current models of analogical comparison have adapted these basic assumptions of SMT (for reviews, see Gentner & Forbus, 2011; Kokinov & French, 2003).

to describe the layered structure of Earth by likening it to the layers of a peach. Several studies show that within-domain, concrete spatial comparisons facilitate spatial learning [5], [11], [43]. For example, young children are successful at learning a non-obvious spatial concept when asked to compare two spatial structures, one of which exemplifies the concept and the other which does not [43]. As another example, children learn novel spatial relations better when they compare two exemplars that depict the relation than when they see the exemplars separately [11]. Most studies on spatial comparison have focused on concepts and examples that are almost entirely spatial. An open question is whether providing a spatial comparison can facilitate learning spatial representations with a strong conceptual component, such as graphs and diagrams, where the spatial representation serves to illustrate concepts that are not themselves spatial. There is reason to think that spatial analogy can encourage conceptual learning; in natural language, space is frequently analogized to abstract domains (e.g., *She was in between jobs*), indicating that spatial analogy can serve as a springboard for abstract, conceptual knowledge [45]. In this work, we begin to address the question of whether spatial comparison can simultaneously confer both spatial and conceptual relational insight. The current studies focus on learning about graphs, a particularly challenging type of spatial representation.

1.3 Graphs: A Complex Relational Task

Successful graph comprehension requires highly sophisticated spatial and conceptual reasoning. Graphs simultaneously convey spatial relations (one line above another) and conceptual relations (A exceeds B) [18]. It is widely accepted that graph comprehension entails at least three major, intertwined component processes [18-20]. First, viewers must encode the visual array and identify the important visuospatial relationships (e.g., a straight line slanting upward). Second, viewers must identify the underlying conceptual relations that those visuospatial relations represent (e.g., an increasing linear relationship between x and y). Finally, viewers must relate those relations to the variables depicted (e.g. a constant increase in carbon dioxide emissions over time). In sum, when one looks at a graph they must be able to simultaneously identify both the spatial and underlying conceptual relations depicted (see [21] for a related claim about diagrammatic representations more generally). Because of this relational complexity, it is not surprising that students of all ages have difficulties understanding graphs [18], [22-30].

Our question was whether analogical comparison—a process that promotes relational learning—would be a useful tool for learning the challenging spatial task of integrating complex graphical representations. In the experiments presented here, we focused on reasoning about stock-and-flow (SF) graphs. Conceptually, a stock is some entity amount that is accumulated over time by inflows and/or depleted by outflows. Stocks can only be changed via these flows. The amount of stock in a system is determined by the relationship between inflow and outflow: when inflow exceeds outflow, the stock will increase; when outflow exceeds inflow, the stock will decrease; and when inflow equals outflow, the stock will stabilize.

Stocks and flows are pervasive across domains—for example, they capture the dynamics of water in a bathtub (Figure 1), cash flow of a bank account, and CO₂ levels in the atmosphere. These stock and flow relations are often depicted graphically, as in Figure 2. SF graph problems, even simple ones, are unintuitive and difficult, even for highly educated people with substantial training in science, technology, engineering, and mathematics (STEM) [23], [29-33].

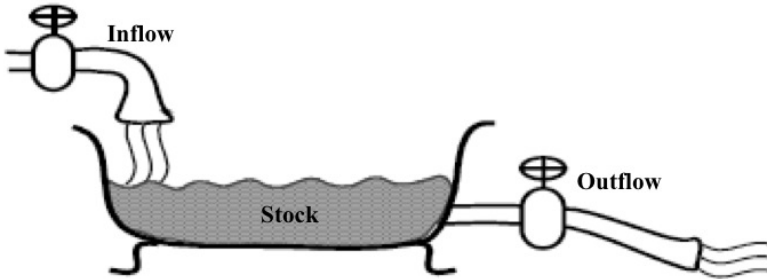


Fig. 1. Stocks and Flows in a bathtub. The amount of water in the tub is the stock. Water entering the tub through the faucet is the inflow. Water leaving the tub through the drain is the outflow.

1.4 The Current Experiments

In this set of studies, we tested whether presenting spatial analogies between graphical systems can help students learn to reason about stock-and-flow graphs like those depicted in Figure 2. The basic idea of our intervention was to juxtapose contrastive graphs and encourage students to compare them. This intervention was based on two principles of comparison processing derived from structure-mapping theory: (1) abstraction: analogical comparison reveals common structure [2-3], [8], [13]; and (2) contrast: analogical comparison highlights alignable differences—differences along a common dimension or predicate that plays the same role in the common structure [15-16].

These principles, taken together, predict that if learners align two analogous but contrasting examples, the common structure will become more salient and any alignable differences will become more noticeable [16]. This prediction has been borne out in studies of relational mapping and transfer in adults [4], [34] and children [5], [35-37], [43], in both conceptual and spatial domains. For example, Gentner et al. [43] found evidence that comparison can help children learn a non-obvious spatial concept, namely that *triangles confer stability in construction*. Specifically, when children were shown two toy buildings, a stable one that contained a triangle and a wobbly one that did not, children could use the alignment between them to identify the distinctive part (the triangle) as important for stability.

A third principle that is particularly relevant for research on learning is that alignment is easier and less error-prone for novice learners (both children and adults) when the items being compared are highly similar in their surface features as well as in their relational structure, i.e., the items are *literally similar* [38-41], [43].

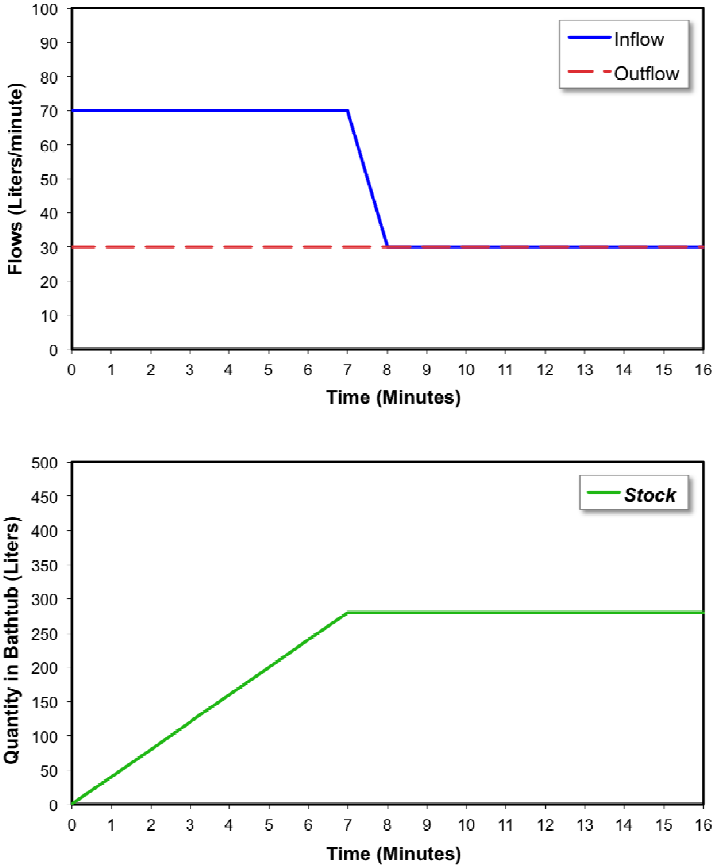


Fig. 2. A typical set of stock and flow graphs. The top graph depicts the changing rates of inflow from the faucet (solid line) and outflow through the drain (dotted line) over time. The line in the bottom graph shows the resulting change in the stock amount, i.e., bathtub water. Notice that as long as inflow exceeds outflow, the stock continues to rise. In contrast, when inflow equals outflow, the stock stabilizes.

Researchers suggest that the literal similarity advantage exists because object similarities support the required relational alignment. For example, in the part-learning study just described, young children (3-year-olds) were far better at aligning the creatures and noticing the contrasting parts when the pairs were highly similar (making them easy to align). Similar results have been obtained with adults. For example, Markman and Gentner [16] found that people list more relational similarities and alignable differences for literally similar scenes than for analogous scenes that contained fewer object- or surface matches. Even in online sentence processing, literal-similarity matches are processed faster than purely relational matches [42].

The studies consisted of a self-paced graph training task, followed by a set of graphical integration problems involving stocks and flows, which are described below. In the first study, we examined whether comparing examples leads to better performance on the graphical integration task than studying the same examples sequentially. In the second study, we varied the similarity of the pairs being compared during training, the details of which we will discuss later.

2 Experiment 1

2.1 Method

Participants. 32 undergraduate students from Northwestern University took part in the study individually or in groups of two. Participants completed the task in 15-25 minutes and for their time they received credit towards a course requirement.

Materials and Procedure. The experimenter gave one task booklet to the participant and upon completion they returned the booklet to the experimenter. The booklet contained a graph-training task followed by a graphical integration test. To make the task more concrete, all graphs were described in the context of CO₂ levels, where the stock was the amount of CO₂ in the atmosphere, inflow was the rate of CO₂ emissions, and outflow was the rate of CO₂ removal from the atmosphere (e.g., as it is taken up by plants).

Graph-Training Task. During the training phase, participants saw three examples of stock and flow graphs, similar to the graphs in Figure 3. To facilitate structural alignment, each example looked exactly the same up to the midpoint of the x-axis (time = 8). After the midpoint the examples differed in which of the three basic relationships between inflow, outflow, and stock they depicted: when inflow exceeded outflow, the stock was increasing; when outflow exceeded inflow, the stock was decreasing; and when inflow was equivalent to outflow, the stock was stable². Thus, each of the examples only differed in one key relation between the three variables. Participants were randomly assigned to the *Sequential* or the *Comparison* training condition. In the Sequential condition, participants saw the three examples on separate pages. After seeing each example, participants were asked to explain the graphs by describing “What is going on in the TOP graph” and also “What is going on in the BOTTOM graph” (emphasis in the original instructions). The order in which the examples were shown was counterbalanced across participants. In the Comparison condition, participants saw two examples side-by-side and were asked to describe both similarities

² There are several more complex relations involving changes in net flow and the shape of the stock graph, but systematically varying those would compound the number of examples to be used. Thus in these studies we only focus on the three most basic relationships between stock and flows.

and differences between the two sets of graphs by listing “What is **similar** about the TOP (BOTTOM) graphs” and “What is **different** about the TOP (BOTTOM) graphs” (emphasis in the original instructions). Participants in the Comparison group only saw two stock-and-flow graph examples at one time; in order to make sure they saw all three examples, we gave them two separate comparisons to make. Thus, the Comparison group saw one of the examples twice (in two different comparison sets). The repeated example and the position of the example on the page (left or right) were counterbalanced across participants.

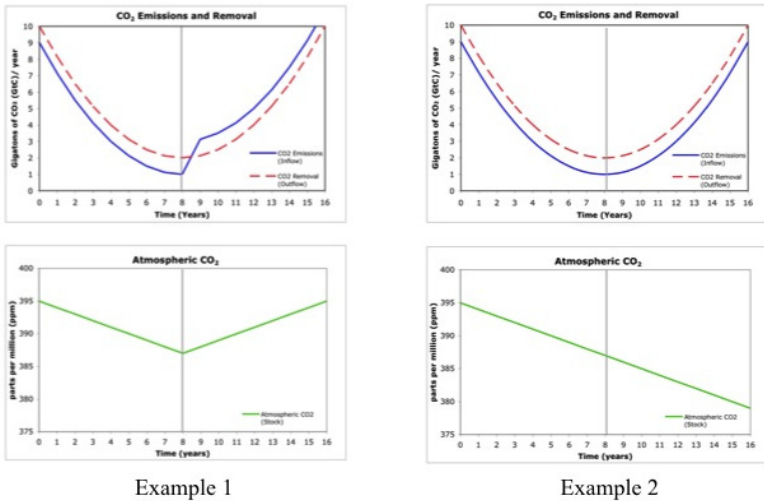
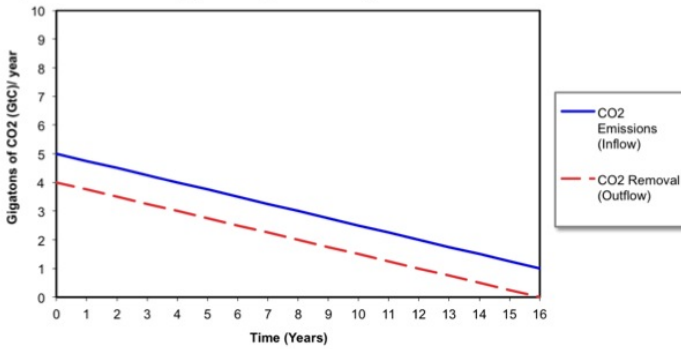


Fig. 3. Sample Comparison Examples. The inflow/outflow (top) graphs are the same until the midpoint, when the inflow (solid line) trajectory changes. Likewise, the stock (bottom) graphs are the same up until the midpoint, when the stock trajectory changes, corresponding to the change in the inflow/outflow graph. In the training task, participants were directed to compare and contrast the top two graphs, and then compare and contrast the bottom two graphs.

Graphical Integration Task. We adapted the *graphical integration* task from Booth Sweeney and Sterman [23]. In their original study, highly educated graduate students were presented with a picture of a bathtub and graphs showing the inflow and outflow of water, then asked to draw the trajectory of the stock of water in the tub. We used similar problems, although they were introduced in the context of CO₂ levels in the atmosphere (Figure 3). Participants solved seven graphical integration problems.

The graph below shows a hypothetical pattern of CO_2 *Emissions* and *Removal*.



On the graph below, draw the pattern of *Atmospheric* CO₂ that would be produced by the Emissions and Removal pattern above. The green dot (•) at time zero shows the initial atmospheric CO₂ level.

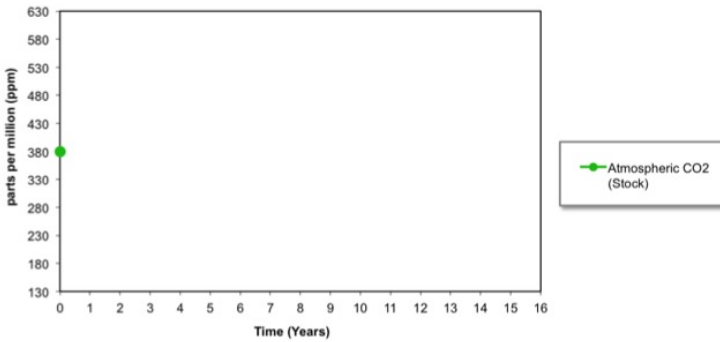


Fig. 4. Sample Graphical Integration problem. Participants were given a graph that depicted inflows and outflows to the stock over time. They had to draw the resultant stock in the bottom graph.

2.2 Measures

Problem Score. For each graphical integration problem, participants received either 0 or 1 point. Participants received one point if their response maintained the three basic relations between stock and flow. For example, if the inflow was greater than outflow from $t=0-8$, then the participant needed to draw a stock that was continually increasing from $t=0-8$. Quantitative inaccuracies were not penalized. Participants could achieve a maximum score of 7 across the seven problems. Two raters blind to condition scored each problem. There was high interrater agreement, (96%, $\kappa = 0.88$); all disagreements were resolved through discussion.

2.3 Results and Discussion

Our prediction was that participants who were given the opportunity to compare contrastive graphs would perform better on the graphical integration problems. This prediction was borne out in the data. Participants who compared examples performed better ($M=4.75$, $SE=0.49$) on the graphical integration test than participants who studied the examples separately ($M=3.32$, $SE=0.78$), $t(30)=2.18$, $p<.05$, $d=0.77$. Why do we see this performance advantage for the Comparison group? We suggest that the act of comparing the graphs enabled people to both (1) identify the relations common to both graphs and (2) notice relational contrasts between them. That is, when learners were given the opportunity to align two analogous but contrasting examples, the common structure became more salient and the alignable differences between the graphs were more noticeable [16]. These two phenomena are exemplified in the similarity/difference listings from two of the Comparison participants:

- “From $t=0-8$ inflow exceeds outflow.” (Similarity)
- “From $t=8-16$ inflow still exceeds outflow in [the top left graph], but inflow is less than outflow in [the top right graph].” (Difference)
- “Both CO_2 contents increase from time 0 to 9” (Similarity)
- “In [the bottom left graph]; total stock CO_2 goes down after 8 yrs. vs [the bottom right] graph where the stock CO_2 value continues to increase.” (Difference)

Our results are consistent with the claim that the structural alignment process both highlights common relational structure and accentuates alignable differences. Furthermore, these data suggest that spatial analogy can facilitate learning about spatial representations with a strong conceptual component. In experiment two, we wanted to test a further prediction of structural alignment models of analogical comparison—namely, comparing examples that share greater overall similarity (i.e., surface and structural similarity) will be more beneficial for learning than comparing examples where there is less surface similarity.

3 Experiment 2

Prior work demonstrates that structural alignment is easier for learners when the items being compared are highly similar in their surface features as well as in their relational structure [38], [43]. The claim is that, in cases of high similarity, surface similarity works in the service of relational similarity, and thus effectively guides learners to the correct alignment. Maximizing the likelihood that a learner achieves a successful structural alignment increases the likelihood that they will notice important relational commonalities and differences. Thus, a greater likelihood of successful alignment should translate into a greater likelihood of successful relational learning. Several studies have demonstrated a learning advantage for high similarity comparisons. In the toy building task mentioned above, children learned better when the two compared buildings shared high surface similarity, in contrast to low surface similarity [43]. Most

studies that report a high similarity advantage in learning by comparison have focused on children's learning [5], [11], [43]; our question is whether we will see a similar advantage for high similarity with adults in a complex arena such as graph integration. In this study, we varied the similarity of examples that participants compared during training. Participants either compared graphs that shared both relational (structural) similarity and surface similarity—i.e., they had high overall similarity—or they compared graphs that shared structural similarity but were perceptually dissimilar—they had low overall similarity.

3.1 Method

Participants. 62 undergraduate students from Northwestern University took part in the study individually or in groups of two. Participants completed the task in 15-25 minutes and for their time they received credit towards a course requirement.

Materials and Procedure. The procedure was as in Experiment 1—an experimenter handed a booklet to the participant. Upon completion the participant gave the booklet back to the experimenter. The booklet contained a graph-training task followed by a graphical integration test.

High Alignment vs. Low Alignment Training. All participants compared examples during training, what differed was the overall similarity between the examples. One group of participants compared example graphs that shared both structural similarity and perceptual similarity. Specifically, the compared graphs contained the same relations between variables. For example, in Figure 3 both of the top graphs show outflow above (exceeding) inflow from $t=0-8$. In addition, the trajectories or shapes of the lines in the graphs were similar; in Figure 3, for example, the outflow line is parabolic in both graphs. These graphs were considered highly alignable because they shared both relational and surface similarity. For the sake of clarity, we call this the *Same Shape* training condition. Another group of participants compared graphs that maintained relational similarity, but were less perceptually similar. Thus, the same relations between inflow, outflow and stock were present (e.g., outflow exceeds inflow), but the shapes of the variable lines were different (e.g., the inflow was a parabolic function in one graph and an exponential function in the other). These graphs were considered less alignable because surface similarity could not facilitate alignment to the same degree. We call this the *Different Shape* condition. Participants were asked to list the similarities and differences for each comparison set, as in Experiment 1.

Graphical Integration Test. The graphical integration test was as in Experiment 1.

Measures

Problem Score. We scored each graphical integration response as in Experiment 1. For each graphical integration problem, participants received either 0 or 1 point, for a maximum of 7 points across seven problems.

3.2 Results

As predicted, participants who compared Same Shape examples performed better on the graphical integration problems ($M=4.35$, $SE=0.40$) than those who compared Different Shape examples ($M=3.32$, $SE=0.47$), $t(60)=1.67$, $p<0.05$, $d=0.42$, one-tailed. Overall, these results are consistent with our prediction that performance is related to the ease of alignment, with students who were exposed to High Alignability (Same Shape) training performing better than those that were exposed to Low Alignability (Different Shape) training. Comparing highly similar graphs enabled people to more easily identify the important relational commonalities and differences between the graphs, as exemplified in one participant's similarity/difference listings:

- “For the first 8 years, the CO₂ removal (outflow) is greater than CO₂ emission inflow” (Similarity)
- “After 8 years, [the left graph] has a greater inflow than outflow while [the right graph] has same amount of inflow and outflow” (Difference)

In contrast, comparing less similar graphs made it more difficult for people to focus on the relevant relational commonalities and contrasts. Below is a representative similarity/difference listing for participants in the Different Shape condition. These participants tended to describe superficial characteristics of the graphs rather than relational aspects.

- “They both measure inflows and outflows of CO₂; they have the same key, and the same axis measurements; same colors; same titles” (Similarity)
- “[The left graph] is smooth; [the right graph] is straight until sharp junction” (Difference)

In sum, we found that pairs that were easier to spatially align (because they were perceptually similar) were more helpful in training, and led to better performance on the graphical integration test, than pairs that were more difficult to align. These results are consistent with the claim that, in early learning, comparing examples that are readily alignable—such as pairs that share overall similarity—is especially beneficial [5], [37], [43].

4 Discussion

These experiments provide initial evidence that the principles of structure-mapping can be used effectively to promote students' learning in a domain with a high degree of relational complexity. Specifically, spatial alignment (spatial analogy) of examples facilitated the sophisticated spatial and conceptual reasoning required for the task. Participants who compared examples of stock-and-flow graphs during training were able to transfer their understanding to graphical integration problems. Our results also support the claim that ease of spatial alignment contributes to graph learning. Participants who saw perceptually similar graphs were better able to align them and notice

the key relational commonalities and differences between the variables on the graphs—e.g., that inflow exceeds outflow. This advantage for ease of spatial alignment is consistent with prior findings on spatial learning [5], [43].

In future work, we aim to further explore variability in the surface and structural similarity between examples. It would also be useful to identify other aspects of graphical examples that may make them easier or harder to align. Another issue that should be explored is how to better facilitate learning via comparison. In our studies, overall performance across conditions was not at ceiling—participants have room to grow in their learning. In addition to exploring the issue of optimal variation in examples, it would also be useful to develop ways to guide the comparison process more effectively. In the above studies, the comparison task was fairly open-ended—people were only asked to describe similarities and differences between the graphs. Prior work has shown that greater scaffolding during the comparison process leads to better learning [44]; it seems likely that constructing a more guided comparison task would be advantageous for helping students hone in on the multitudinous and complex relations embedded within graphs.

Overall, our findings offer evidence that spatial analogical alignment can be used effectively for graph learning. In our study, detailed predictions from structure-mapping theory and research were found to be applicable for promoting students' graphical learning and reasoning.

Acknowledgements. This research was supported by NSF grant SBE-0541957, the Spatial Intelligence and Learning Center (SILC). We thank Garrett Honke and the Cognition and Language Lab for help with this research.

References

1. Gentner, D.: The development of relational category knowledge. In: Gershkoff-Stowe, L., Rakison, D.H. (eds.) *Building Object Categories in Developmental Time*, pp. 245–275. Erlbaum, Hillsdale (2005)
2. Namy, L.L., Gentner, D.: Making a silk purse out of two sow's ears: Young children's use of comparison in category learning. *J. Experimental Psychology: General* 131(1), 5–15 (2002)
3. Star, J.R., Rittle-Johnson, B.: It pays to compare: An experimental study on computational estimation. *J. Experimental Child Psychology* 101, 408–426 (2009)
4. Gentner, D., Loewenstein, J., Thompson, L.: Learning and transfer: A general role for analogical encoding. *J. of Educational Psychology* 95(2), 393–405 (2003)
5. Loewenstein, J., Gentner, D.: Spatial mapping in preschoolers: Close comparisons facilitate far mappings. *J. of Cognition and Development* 2(2), 189–219 (2001)
6. Gentner, D.: Structure-mapping: A theoretical framework for analogy. *Cognitive Science* 7(2), 155–170 (1983)
7. Gentner, D., Markman, A.B.: Structure mapping in analogy and similarity. *American Psychologist* 52, 45–56 (1997)
8. Gick, M.L., Holyoak, K.J.: Schema induction and analogical transfer. *Cognitive Psychology* 15(1), 1–38 (1983)

9. Gentner, D.: Why we're so smart. In: Gentner, D., Goldin-Meadow, S. (eds.) *Language in Mind: Advances in the Study of Language and Thought*, pp. 195–235. MIT Press, Cambridge (2003)
10. Gentner, D.: Bootstrapping children's learning: Analogical processes and symbol systems. *Cognitive Science* 34(5), 752–775 (2010)
11. Christie, S., Gentner, D.: Where hypotheses come from: Learning new relations by structural alignment. *J. Cognition and Development* 11(3), 356–373 (2010)
12. Gentner, D., Medina, J.: Similarity and the development of rules. *Cognition* 65, 263–297 (1998)
13. Markman, A.B., Gentner, D.: Structural alignment during similarity comparisons. *Cognitive Psychology* 25, 431–467 (1993)
14. Gentner, D., Gunn, V.: Structural alignment facilitates the noticing of differences. *Memory and Cognition* 29(4), 565–577 (2001)
15. Gentner, D., Markman, A.B.: Structural alignment in comparison: No difference without similarity. *Psychological Science* 5(3), 152–158 (1994)
16. Markman, A.B., Gentner, D.: Splitting the differences: A structural alignment view of similarity. *J. Memory and Language* 32, 517–535 (1993)
17. Gentner, D., Sagi, E.: Does “different” imply a difference? A comparison of two tasks. In: Sun, R., Miyake, N. (eds.) *Proceedings of the Twenty-Eighth Annual Conference of the Cognitive Science Society*, pp. 261–266. Erlbaum, Mahwah (2006)
18. Carpenter, P., Shah, P.: A model of the perceptual and conceptual processes in graph comprehension. *J. Experimental Psychology: Applied* 4, 75–100 (1998)
19. Tversky, B.: Spatial schemas in depictions. In: Gattis, M. (ed.) *Spatial Schemas and Abstract Thought*, pp. 79–111. MIT Press, Cambridge (2001)
20. Pinker, S.: A theory of graph comprehension. In: Freedle, R. (ed.) *Artificial Intelligence and the Future of Testing*, pp. 73–126. Erlbaum, Hillsdale (1990)
21. Gattis, M.: Mapping relational structure in spatial reasoning. *Cognitive Science* 28(4), 589–610 (2004)
22. Bell, A., Janvier, C.: The interpretation of graphs representing situations. *For the Learning of Mathematics* 2(1), 34–42 (1981)
23. Booth Sweeney, L., Sterman, J.D.: Bathtub Dynamics: Initial Results of a Systems Thinking Inventory. *System Dynamics Review* 16, 249–294 (2001)
24. Culbertson, H.M., Powers, R.D.: A study of graph comprehension difficulties. *AV Communication Review* 7, 97–110 (1959)
25. Gattis, M., Holyoak, K.J.: Mapping conceptual to spatial relations in visual reasoning. *J. Experimental Psychology: Learning, Memory, and Cognition* 22, 231–239 (1996)
26. Kozhevnikov, M., Hegarty, M., Mayer, R.E.: Revising the visualizer/verbalizer dimension: Evidence for two types of visualizers. *Cognition and Instruction* 20, 47–77 (2002)
27. Maichle, U.: Cognitive processes in understanding line graphs. In: Schnotz, W., Kulhavy, R.W. (eds.) *Comprehension of Graphics*, pp. 207–226. Elsevier Science, New York (1994)
28. Shah, P., Carpenter, P.A.: Conceptual limitations in comprehending line graphs. *J. Experimental Psychology: General* 124, 43–61 (1995)
29. Sterman, J.D., Booth Sweeney, L.: Cloudy Skies: Assessing Public Understanding of Global Warming. *System Dynamics Review* 18, 207–240 (2002)
30. Sterman, J.D., Booth Sweeney, L.: Understanding Public Complacency About Climate Change: Adults' Mental Models of Climate Change Violate Conservation of Matter. *Climatic Change* 80, 213–238 (2007)

31. Cronin, M.A., Gonzalez, C., Sterman, J.D.: Why don't well-educated adults understand accumulation? A challenge to researchers, educators, and citizens. *Organizational Behavior and Human Decision Processes* 108, 116–130 (2009)
32. Cronin, M., Gonzalez, C.: Understanding the building blocks of system dynamics. *System Dynamics Review* 23(1), 1–17 (2007)
33. Pala, Ö., Vennix, J.A.M.: Effect of system dynamics education on systems thinking inventory task performance. *System Dynamics Review* 21(2), 147–172 (2005)
34. Catrambone, R., Holyoak, K.J.: Overcoming contextual limitations on problem-solving transfer. *J. Experimental Psychology: Learning, Memory, and Cognition* 15(6), 1147–1156 (1989)
35. Gentner, D., Namy, L.: Comparison in the development of categories. *Cognitive Development* 14, 487–513 (1999)
36. Mutafchieva, M., Kokinov, B.: Does the family analogy help young children to do relational mapping? In: *Proceedings of the European Conference on Cognitive Science*, pp. 407–412. Erlbaum, Hillsdale (2007)
37. Gentner, D., Loewenstein, J., Hung, B.: Comparison facilitates children's learning of names for parts. *J. Cognition and Development* 8, 285–307 (2007)
38. Gentner, D., Ratterman, M.J., Forbus, K.D.: The roles of similarity in transfer: Separating retrievability from inferential soundness. *Cognitive Psychology* 25, 524–575 (1993)
39. Gentner, D., Toupin, C.: Systematicity and surface similarity in the development of analogy. *Cognitive Science* 10, 277–300 (1986)
40. Paik, J.H., Mix, K.S.: Preschooler's use of surface similarity in object comparisons: Taking context into account. *J. Experimental Child Psychology* 95(3), 194–214 (2006)
41. Richland, L.E., Morrison, R.G., Holyoak, K.J.: Children's Development of Analogical Reasoning: Insights from Scene Analogy Problems. *J. Experimental Child Psychology* 94, 246–273 (2006)
42. Gentner, D., Kurtz, K.: Relations, objects, and the composition of analogies. *Cognitive Science* 30, 609–642 (2006)
43. Gentner, D., Levine, S., Dhillon, S., Poltermann, A.: Using structural alignment to facilitate learning of spatial concepts in an informal setting. In: Kokinov, B., Holyoak, K.J., Gentner, D. (eds.) *Proceedings of the Second International Conference on Analogy*. NBU Press, Sofia (2009)
44. Kurtz, K.J., Miao, C., Gentner, D.: Learning by analogical bootstrapping. *J. Learning Sciences* 10(4), 417–446 (2001)
45. Lakoff, G., Johnson, M.: *Metaphors We Live By*. University of Chicago Press, Chicago (1980)