Research Report

SIMILARITY INVOLVING ATTRIBUTES AND RELATIONS: JUDGMENTS OF SIMILARITY AND DIFFERENCE ARE NOT INVERSES

Douglas L. Medin, Robert L. Goldstone, and Dedre Gentner
Department of Psychology, University of Illinois

Abstract—*Conventional wisdom and previous research suggest that similarity judgments and difference judgments are inverses of one another. An exception to this rule arises when both relational similarity and attributional similarity are considered. When presented with choices that are relationally or attributionally similar to a standard, human subjects tend to pick the relationally similar choice as more similar to the standard* and *as more different from the standard. These results not only reinforce the general distinction between attributes and relations but also show that attributes and relations are dynamically distinct in the processes that give rise to similarity and difference judgments.*

The question of what makes things seem alike or seem different is fundamental to cognition. Models of learning imply that the learning of a task is facilitated if it is similar to another task that is already part of a learner's repertoire (Thorndike, 1966). Stimulus generalization occurs as a function of how similar the new stimulus is to the conditioned stimulus (Pavlov, 1927; Shepard, 1987). In memory models it has been assumed that X reminds people of Y if it is similar to Y (Kolodner, 1984; Schank, 1982), and in theories of categorization it has been assumed that classification of new examples is based on their similarity to known examples or to a category prototype (Medin & Smith, 1984; Oden, 1987). In short, similarity is a central concept in cognitive science.

Correspondence and reprint requests to: Douglas L. Medin, Dept. of Psychology, University of Michigan, 330 Packard Rd., Ann Arbor, MI 48103.

Much of the research and theorizing about similarity has been organized around two tenets that on initial consideration seem to be self-evident. The first is that similarity and difference judgments are logical opposites. This intuition is quite compelling. For example, given the set of similarity pairs dog/wolf, dog/cat and dog/teacup, the most similar pair (dog/wolf) is the least different, and the most different (dog/teacup) is the least similar. The Scottish philosopher James Mill stated that "distinguishing differences and similarities is the same thing; a similarity being nothing but a slight difference" (Mill, 1829, pp. 13-14). The second tenet is that the underlying basis for similarity is shared properties or parts. In the words of psychologist and philosopher William James, "To abstract the *ground* of either difference or likeness (where it is not ultimate) demands an analysis of the given objects into their parts" (1890, p. 529). That is to say, similarity is based on partial identities or shared constituents. The results to be reported in this paper call both of these tenets into question. We argue that accounts of similarity need to include not only shared properties or attributes but also relations among properties. Further, our experiments show that attributes and relations behave differently in similarity judgments versus difference judgments, with the consequence that these two types of judgments are not complementary. Before describing these results, however, we need to provide a bit more by way of background.

SIMILARITY VERSUS DIFFERENCE

If judgments of similarity and difference were complementary measures of the same psychological facts, then judged difference should be a linear function of judged similarity with a slope of −1. This basic notion has received clear support. For example, in one study comparing judgments of similarity and difference of lowercase letters (Hosman & Kuennapas, 1972), the correlation between the judgments was −.98 and the slope of the regression line was −.91. Tversky (1977) reported a correlation of −.98 when people judged the similarity or difference of 21 pairs of countries using a 20-point rating scale.

Similarity and difference judgments mirror each other so closely that they tend to be used interchangeably in multidimensional scaling studies where the goal is to represent similarity in terms of distance in some psychological space (Shepard, 1974). When direct comparisons are made, the scaling solutions tend to be virtually identical for similarity and dissimilarity judgments (Rapoport & Fillenbaum, 1972).

One note of discord is an experiment reported by Hollingworth (1913). He asked people to rank samples of handwriting with respect to their similarity or difference from a standard. Individual participants ranked similarities on two occasions and differences on two occasions. Hollingworth found that the degree of correlation between the two similarity judgments and between the two difference judgments was greater than the degree of correlation between similarity judgments and difference judgments. On the basis of this disparity in correlation and on participants' introspections, Hollingworth suggested that difference judgments tend to be based on fine details whereas similarity judgments are base on "gross and more general criteria."

Hollingworth's results have had little impact, partly because the study does not meet modern standards with respect to experimental design. For example, although the four sets of judgments were made at least one week apart, the time between judgments of the same type was less than that between judgments of opposite types. In addition, the introspective reports given in Hollingworth's paper do not appear to support his summary. Finally, there was an average correlation of −.92 between the similarity and difference rankings, quite consistent with the other cited studies.

Aside from Hollingworth's results, the only important exception to the well-established inverse relation between similarity and difference is a result reported by Tversky (1977) that applies when pairs of objects differing greatly in amount of detail (number of features) are compared. This exception is best understood in terms of his contrast model. The contrast model assumes that similarity between two entities is a weighted function of matching and mismatching features. The contrast model can be described in terms of the following equation:

$$S_{(A,B)} = \theta f(A \cap B) - \alpha f(A - B) - \beta f(B - A). \quad (1)$$

The similarity of A to $B, S_{(A,B)}$, is a weighted function of features that A and B share, $f(A \cap B)$, features present in A but not $B, f(A - B)$, and features present in B but not $A, f(B - A)$. The parameters θ, α, and β are weighting coefficients. These weights, however, need not be fixed but rather may vary with the context and judgment task. In particular, Tversky has found support for the conjecture that common features are weighted more heavily (relative to mismatching features) in judgments of similarity than in judgments of difference. If this is true, then a pair of objects with many common and many distinctive features may be judged to be both more similar and more different than another pair of objects with fewer common and fewer distinctive features. In one test of this hypothesis, subjects were given sets of two pairs of countries and asked to pick either the pair that was more similar or the pair that was more different in each set. The pairs of countries differed in how well-known they were to participants (e.g., East Germany–West Germany vs. Ceylon–Nepal). Tversky found that the more well-known or prominent pair of countries tended to be selected as both more similar and more different. Collapsing across the two types of judgments, prominent pairs were selected an average of 56.8% of the time, significantly more often than the 50% expected if similarity and difference judgments were complementary. The results are consistent with the contrast model. They also support the conclusion that similarity and difference judgments need not be inverses when entities differ in their number of features. In this paper we propose an important second exception, based on the distinction between attributes and relations.

ATTRIBUTES VERSUS RELATIONS

We use *attribute* to refer to any constituent property of a stimulus. Relative to a particular representation system, an attribute is a primitive if it is not decomposable into other terms (attributes) in that system. We use *relations* to refer to descriptions of connections between two or more objects or attributes. Relations can take objects, attributes, or other relations as arguments (Palmer, 1978; Gentner, 1983). For example, suppose that stimulus A consists of a red square and a red circle and that stimulus B consists of a blue circle and a blue triangle. The A and B stimuli would share the attribute "circle" and the relation "same color." Note that this distinction between attributes and relations is relative to a psychological representation. Logically, it is possible to define a given feature as either an attribute or relation, and there is an indefinite number of properties and relations that in principle apply to any entity (Goodman, 1972). Our claim is that people construe situations in terms of attributes of objects and relations between objects, and that this distinction is psychologically salient.

The distinction between attributes and relations is not inconsistent with Tversky's featural contrast theory, since one might treat relations simply as additional features entering into the computation of similarity. In practice, however, we believe that psychological theories of similarity need to keep attributes and relations distinct. One salient reason for this belief is our observation that attributes and relations do not behave in the same way for difference judgments as they do for similarity judgments.

We do not wish to imply that the distinction between attributes and relations is novel. In both artificial intelligence and psychological approaches to visual perception, structural descriptions that include parts, relations among parts, and even relations among relations are common (Biederman, 1987; Marr, 1982; Norman & Rumelhart, 1975; Palmer, 1975, 1978; Winston, 1975). What is novel is the evidence that the relative weighting of attributes and relations depends on the judgment task, specifically on whether similarity or dissimilarity judgments are involved.

EXPERIMENTS

Methods

The first study used 36 sets of geometric stimuli like those shown in Figure 1. On each trial, the display consisted of a standard (T) and two choice stimuli (A and B), and participants judged either which alternative was more similar to the standard or which alternative was more different from the standard. One alternative always shared a unique attribute with the standard (i.e., one that the other did not possess); the other shared a unique relation with the standard. For example, in Figure 1, choice A matches the standard in the shading attribute of the circle and choice B matches in the relation of all forms having the same shading. The left-right position of the choice stimuli and the order in which examples were seen were counter-balanced across subjects. Altogether, 23 subjects made similarity judgments and 45 subjects made difference judgments. The particular attributes and relations employed varied across stimulus sets. The relations used were: *same shape, same color, same height, same orientation, larger than,* and *greater number than.* The attributes used were *shading* (e.g., striped), *shape* (e.g., square), *size* (e.g.,

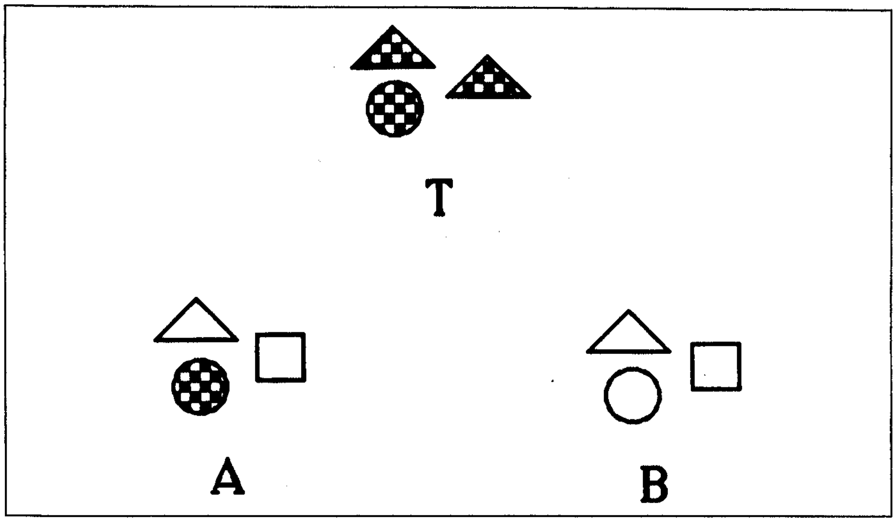

Fig. 1. Sample stimulus: T, the standard, is attributionally similar to A because they both have a checkered circle. B does not contain this attributional similarity to T; instead, B has a matching relation, "same-shading," with T.

large), and *orientation* (e.g., upward pointing).

The second study used properties integrated to form coherent, meaningful patterns,—cartoon-like butterflies (see Figure 2). The procedure was again forced choice—which choice stimulus is more similar to (more different from) the standard—with one alternative having an extra matching attribute and the other having an extra matching relation. For example, in Figure 2 the B alternative has a left wing that matches the standard and the A alternative has the shared relation "right wing larger than left wing." Altogether 54 people made similarity judgments and 34 people made difference judgments. Seventy triads of stimuli were employed, consisting of approximately equal numbers of each of the following relations: *same color, different color, darker than, lighter than, same size, smaller than,* and *larger than.*

Results

In both studies similarity and difference judgments show clear departures from complementarity. In particular, the relative importance of attributes and relations shifts substantially depending on whether similarity or difference judgments are being made, with relations being more attended to in similarity judgments and attributes more attended to in difference judgments. As a consequence, the alternative involving the extra matching relation tends to be selected as the choice *both* most similar to and most different from the standard. Referring again to Figure 1, people asked to pick the more *similar* alternative choose B, apparently because it shares the relation that the components all have the same coloring. People asked to pick the most *different* alternative also choose B, apparently because it has fewer component-color matches with the target than A does (the shading of each of its three components mismatches the standard whereas only two components for the A choice mismatch in shading).

A straightforward way to summarize these results is to plot the proportion of times the alternative with the extra relational match was picked on similarity trials against the proportion of times it was picked on difference trials. If similarity and dissimilarity judgments are inverses then the points should lie along (both sides of) the negative diagonal. Figure 3 gives this plot for the geometric shapes and Figure 4 shows it for the butterfly stimuli. Almost all of the points lie above the negative diagonal, revealing the trend for the relational choice to be picked for both similarity and difference judgments. This pattern of results held across each of the various relations employed. Including both similarity and difference trials, the relational choice was made 58% of the time for the geometric stimuli and 59% of the time for the butterfly stimuli. These departures from 50% are highly reliable (for the geometric stimuli, $t = 3.89$, $df = 87$, $p < .001$; for the butterfly stimuli, $t = 5.87$, $df = 87$, $p < .001$).

As a further means of evaluating the contribution of attributes versus relations to judgments, multiple regressions were performed with similarity judgment and type of similarity (attributional versus relational) as predictors and difference judgments as the dependent variable. For the geometric stimuli, these factors accounted for 79% of the variance, and both similarity judgment and similarity type were highly significant ($t = 16.69$ and $t = 10.4$, respectively) predictors of difference judgment. For the butterfly stimuli, 66% of the variance was accounted for and again both judgment and similarity type were highly significant predictors ($t = 16.9$ and $t = 9.7$ respectively).

Similarity and difference judgments were negatively correlated, as Figures 3 and 4 suggest. For the geometric stimuli the Pearson r was $-.70$ and for the butterfly stimuli it was $-.67$. When the effect of similarity judgment is partialed out, then difference judgments and number of relation matches are positively correlated ($r = .77$ for the geometric stimuli and $r = .61$ for the butterfly stimuli). The corresponding correlations between similarity judgments and number of relation matches, with difference judgments partialled out, are positive also, with $r = .84$ and $r = .83$, respectively. Again this reveals the systemic pattern of the relational alternative being selected for both similarity and dissimilarity judgments.

The obtained results cannot be explained by hypothesizing that difference judgments show less extreme fluctuations than do similarity judgments. The two plots show that there are triads in which the relational choice is picked as similar approximately 50% of the time for similarity judgments, but is picked more than 70% of the time as more different. Other triads display the opposite trend. Furthermore, similarity and difference judgments show disparate patterns of influence; one choice may be clearly more different from the standard, yet neither choice will be clearly more similar to the standard.

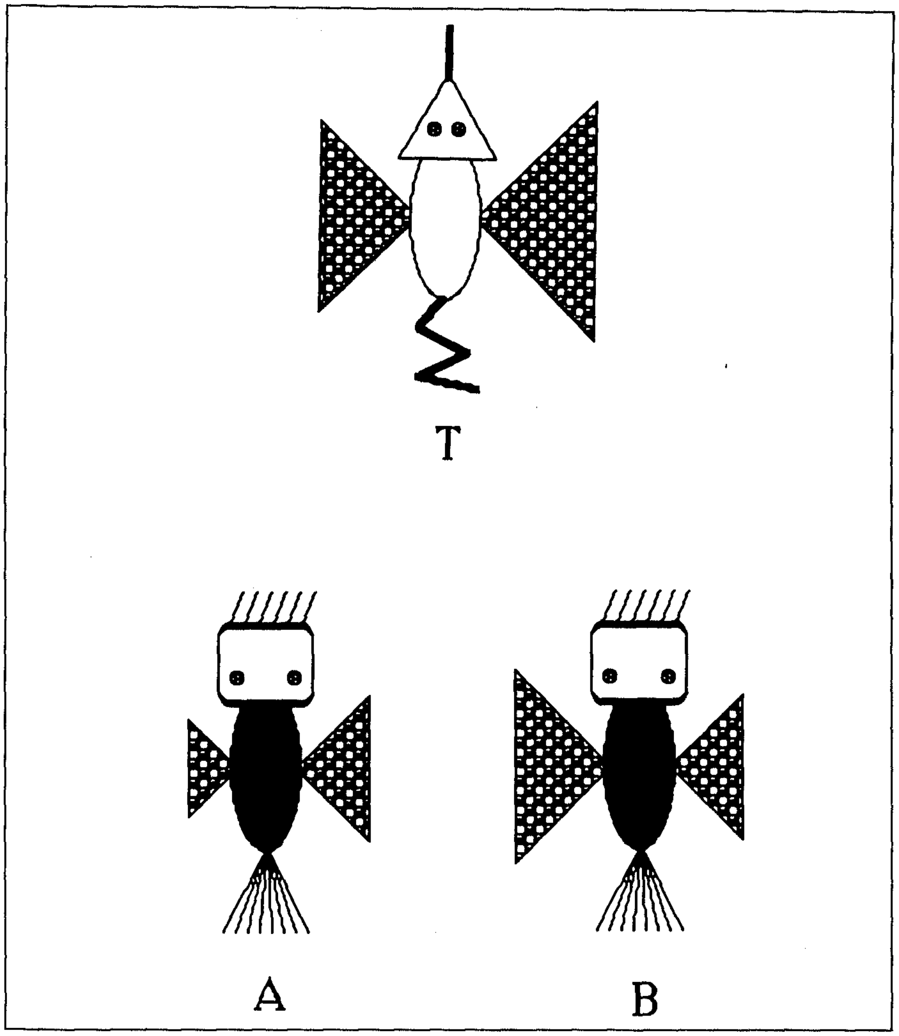

Fig. 2. Sample stimulus: Butterfly T is relationally similar to A, because in both cases the left wing is smaller than the right wing. T is attributionally more similar to B because the left wings of the two butterflies are the same size.

Another possibility that can be rejected is that performance is a mixture of appropriate judgments and a bias toward selecting the relational choice independent of what the target is (e.g. people might prefer Stimulus B over Stimulus A in Figure 1). The reason this interpretation fails is that the relational choice is defined by the target or standard, and in the second study shifted depending on the relation present in the target. Thus in Figure 2, A is the relational choice, but other trials were given with the same choices where the relation in the target was "right wing smaller than left wing." On these trials, B is the relational choice. Therefore a preference for particular stimuli cannot explain our results.

DISCUSSION

Our studies show a new, robust exception to the tendency for similarity and difference judgments to mirror each other. The results are remarkably consistent across the sets of geometric figures and the more naturalistic figures (the butterflies). The alternative with an extra relational match to the standard is more likely to be chosen as both more similar *and* more different than the alternative that shares an extra attributional match with the standard.

It is important to realize that selecting an alternative as more similar is not attributable to variability in the perceptual or judgment process. Variability can lead to an alternative being selected as both more similar and as more different (Ennis, Mullen, Frijters, & Tindall, in press) but variability will not, by itself, lead to an overall pattern of nonmirroring.

Nor can our results be explained by shifts in the weights of matching versus mismatching features. If one assumes that the weighting function for common and distinctive features is independent and additive at the level of features, then an equation analogous to Tversky's contrast model can be developed. Considering again the stimuli in Figure 1 one can describe the difference in similarity of choice B to the target and choice A to the target as:

$$S_{(T,B)} - S_{(T,A)} = \theta(R_s - A_c) - \alpha(A_c - R_s) - \beta(A_u - R_d), \quad (2)$$

where A_c is the feature of being checkered, A_u is the feature of not being checkered, R_s is the relation of same shading, and R_d is the relation of different shading. For the B choice to be picked both as more similar and more dissimilar than the A choice, each side of Equation 2 must be positive for similarity judgments and negative for dissimilarity judgments, while only θ, α, and β are allowed to vary. Assuming that the attributes are properly counterbalanced one would expect that A_c would be equal to A_u. If R_s is also equal to R_d, then the right side of Equation 2 reduces to $(\theta + \alpha + \beta)(R - A)$ and no adjustment of weights will lead to a predicted shift in choices.

Apparently the only plausible way to bring Equation 2 into line with our data is to let R_d be much smaller than R_s. If R_d is equal to zero (the relational difference receives no weight), then choice *A* will be more similar if $R_s > A$ and $\theta + \alpha$ are greater than β. Choice *A* will be more dissimilar when $A(\theta + \alpha + \beta)$ is greater than $(\theta + \alpha)R_s$, which may happen even when R_s is greater than *A* if β is sufficiently large. There are two major problems with this attempted reconciliation. Although it may be plausible to argue that the absence of a relation (e.g., not same shading) is not encoded, many of the relations we employed, such as larger than, more numerous than, and darker than, involve two distinct values rather than the presence or absence of a

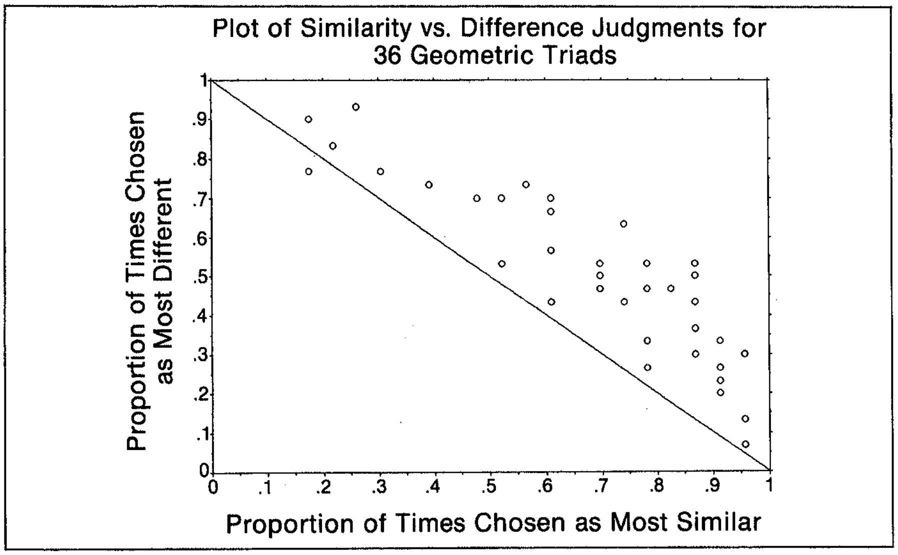

Fig. 3. Only the proportions of relational choices are shown. The fact that the data points lie above the diagonal line indicates that relational pictures tend to be picked as most different and as most similar to the standard.

relation. For example, in Figure 2 the relation in the target was right wing larger than left wing, and the B choice has its left wing larger than the right wing. It would be implausible to argue that the former but not the latter was encoded. Therefore, in general one would expect R_d to be equal in salience to R_s. One could simply assume that, for reasons that are unclear at present, R_d is very small relative to R_s. The main problem with this conjecture is that it amounts to restating our results, namely that relational mismatches contribute less than attributional mismatches to dissimilarity.

The detailed interpretation of the above results is far from clear. Logically, our pattern of results could be produced by either or both of the following: (a) attributional matches are less important than relational matches in similarity judgment, and (b) attributional mismatches are more important than relational mismatches for dissimilarity judgments. The experimental design does not discriminate between these possibilities.

Other research is also consistent with a context-sensitive weighting of properties. For example, Tversky and Gati (1978) found that the influence of matching versus mismatching features in similarity judgments depends on stimulus materials. They found that for pictures, distinctive features were weighted more heavily than common features but that for verbal description of these same pictures, common features were more heavily weighted. This observation, combined with Tversky's earlier observation that the weight of common and distinctive features varies with the type of judgment required, underlines the point that similarity judgments are not tightly fixed but rather vary dynamically with the processing principles associated with a judgment task. That is to say, we do not observe pure effects of structure but rather the output of structure with process.

The fact that processing principles are dynamic provides important clues to similarity structure. *The fact that the relative importance of attribute and relation matches and mismatches depends on whether similarity or dissimilarity judgments are made suggests that attributes and relations are psychologically distinct.*

Our results support cognitive models that draw a sharp distinction between attributes and relations. For example, Gentner's (1983) structure mapping theory of analogy assumes that people focus selectively on relational commonalities in interpreting analogy (Gentner & Clement, 1988). Further, although both attributes and relations influence access to a potential analogy, people making judgments of the "goodness" of an analogy attend selectively to relational properties (Gentner & Landers, 1985).

The present results are not without precedent. There has been extensive research on speeded sameness and difference judgments (Nickerson, 1972). It has been suggested that difference judgments are more analytical than sameness judgments (Hock, 1973). Although sameness judgments are typically described as more global or nonanalytic than difference judgments, an alternative possibility is that they focus on relations rather than attributes (see also Kosslyn, 1987).

The asymmetry between similarity and difference invites a reconsideration

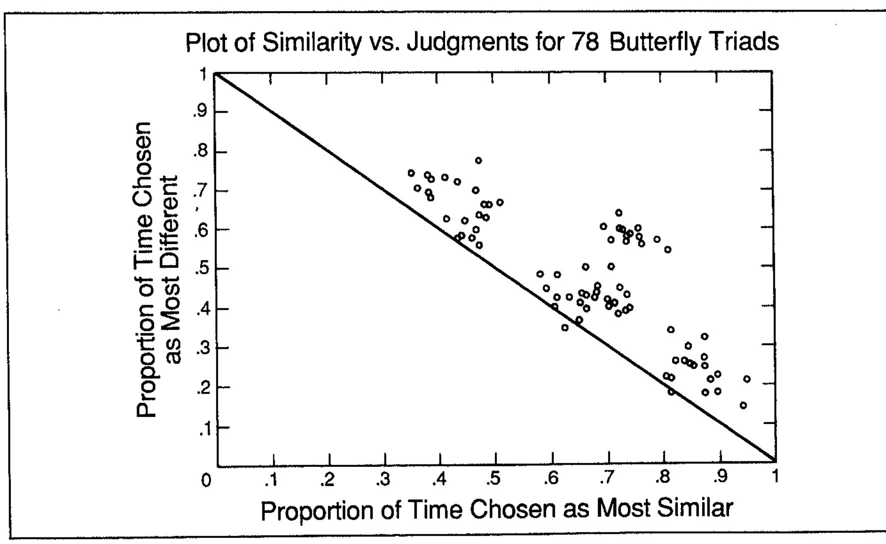

Fig. 4. Only the proportions of relational choices are shown. If similarity and difference judgments are simple inverses, the data points should lie on the diagonal line, not above it.

of data collected under the rubric of mental similarity spaces, which rests on the assumption of complementarity. Similarity and difference data ought not be treated as interchangeable, and scaling solutions may differ depending on which type of judgment has been collected. Furthermore, it may be of considerable importance whether a natural comparison is one of similarity or of difference. More importantly, the contrast between similarity and difference judgments may provide a valuable tool for teasing apart different aspects of cognitive structure.

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